Topology and Symmetry Breaking in ABC Trilayer Graphene

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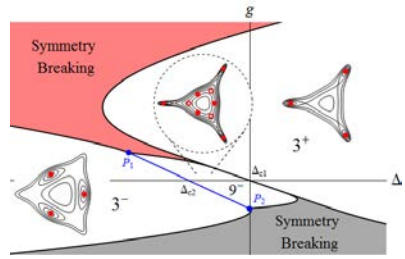
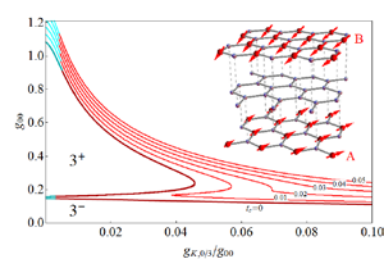
Introduction/Abstract

Rhombohedrally (or ABC) stacked trilayer graphene at half filling has a dispersion consisting of three or nine Dirac cones. The method we developed earlier to study effects of interactions in bilayer graphene [1] seems suited for ABC stacked graphene, too. However, in this work we illuminate a key difference between bilayer and trilayer graphene, which is based on the symmetry. The states at $\pm\mathbf{K}=\pm 4\pi/(3\ 3^{1/2}\ a)$ in bilayer graphene belong to the same irreducible representation (IR) and are therefore guaranteed to be degenerate. On the other hand, in ABC stacked trilayer graphene, the states at $\pm\mathbf{K}$ belong to different IR's and they are split by interlayer hopping process. We perform classification according to the space group of ABC trilayer graphene and find the most general form for the low-energy effective theory which is then related to the microscopically derived model. In the case of non-interacting system we recognize two phase transitions (3rd and 2nd order) driven by the interlayer hopping value (**Figure 1**). The renormalization group (RG) flow equations for all symmetry allowed couplings (15 independent) are derived. At one-loop level the interlayer hopping acquires an anomalous piece in its scaling function which leads to a self-energy effect, not observed in bilayer graphene. We are able to solve the flow equations exactly in the asymptotic limit and we find twelve different stable rays for the RG runaway flow. The phase corresponding to each stable ray is identified. The RG flow equations are solved numerically and we find that, in the case of forward scattering only, the leading instability is toward a mirror breaking gapless state. This state is stable to addition of finite back scattering, however, we find that relatively weak back scattering (**Figure 2**) is sufficient to change the leading instability to anti-ferromagnet or quantum spin Hall state, depending on the sign of the back scattering (repulsive vs attractive, respectively). This seems to be consistent with experiments, where a gapped state (anti-ferromagnet) is observed.



Results and Conclusions

In this work we present the effective low energy theory for ABC stacked trilayer graphene which stems from symmetry analysis. We identify three symmetry indistinguishable but topologically distinct non-interacting phases separated by a 3rd and a 2nd order transition. The value of interlayer hopping δ is the tuning parameter between the three phases. The interaction terms are classified according to the symmetry, and we find 15 independent couplings. These are treated using the coherent state path-integral formalism at finite temperature developed in Ref. [2]. The flow of susceptibilities is used to determine the symmetry breaking channel. One novelty, not present in Ref. [2], is that δ, a term in the non-interacting part of the action, has an anomalous term in its flow equation. This is a self-energy effect with the bare value of δ being renormalized renormalized by the interaction. One consequence is the skew in the phase diagram in Figure 1. At low temperatures δ is the most relevant parameter and it determines the asymptotic behavior of the RG flows. We were able to solve these exactly and find 12 stable fixed rays. For each ray, only a single susceptibility is diverging. We solve the RG equation numerically and find that forward scattering only interaction leads to a mirror breaking gapless phase. This phase is stable to addition of small back scattering, however, as soon as the value of back scattering is of order of 0.005 of forward scattering, a gapped phase, anti-ferromagnet or quantum spin Hall, is preferred (Figure 2). We also numerically find layer polarized, s_{++}-superconducting, and layer polarization density wave state. We study the Hubbard model on trilayer graphene lattice and show that the SO(4) symmetry of this model is preserved. Therefore, we predict that layer polarized and s_{++}-superconducting state have the same susceptibilities and numerically determine these to be the instabilities in the case of attractive Hubbard model. The repulsive Hubbard interaction leads to anti-ferromagnet. The fixed point of the attractive Hubbard model is not stable, hence any infinitesimal interaction that breaks the SO(4) symmetry favors either layer polarized or s_{++}-superconducting state depending on the sign of that term.



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References

[1] Vladimir Cvetkovic, Robert E. Throckmorton, and Oskar Vafek, Phys. Rev. B **86**, 075467 (2012) [24 pages].
[2] Vladimir Cvetkovic, and Oskar Vafek, arXiv:1210.4923.