Quantum Circuits for Measuring Levin-Wen Operators

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Introduction

Recent developments in the theory of quantum computation have shown that the error threshold–a measure of the required qubit coherence and quantum gate accuracy needed to carry out arbitrary quantum computations–may be on the order of ~1%, a significantly less demanding requirement than earlier estimates of 1 part in 10^5, and close to the accuracy obtained experimentally for a number of physical qubit realizations [1]. This improved threshold is obtained using so-called "surface codes" [2], quantum error-correcting codes which can be viewed as the topologically-ordered ground states of a class of 2D lattice models first introduced by Kitaev [3]. Recently Koenig, Kuperberg, and Reichardt [4] have outlined a scheme for performing quantum computation using "non-Abelian" surface codes. These codes are ground states of Levin-Wen models [5], generalizations of the Kitaev model for which the excitations are anyons obeying so-called non-Abelian statistics which can be used to carry out arbitrary quantum computation purely by braiding, (something which is not possible using the "Abelian" Kitaev surface code).

Results and Discussion

Motivated by these developments, we have constructed explicit quantum circuits using standard elements (single qubit rotations, CNOT gates and Toffoli gates) for measuring the commuting set of vertex and plaquette operators that appear in the Levin-Wen model for the case of Fibonacci anyons–the simplest non-Abelian anyons for which universal quantum computation can be carried out by braiding [6]. Such measurements will be required in order to detect errors in the quantum error-correcting code defined by the ground states of this model. We quantify the complexity of these circuits with gate counts using different universal gate sets and find these measurements become significantly easier to perform if n-qubit Toffoli gates with n = 3,4 and 5 can be carried out directly. In addition to measurement circuits, we construct related quantum circuits requiring only a few qubits which can feasibly be carried out experimentally using existing qubits.

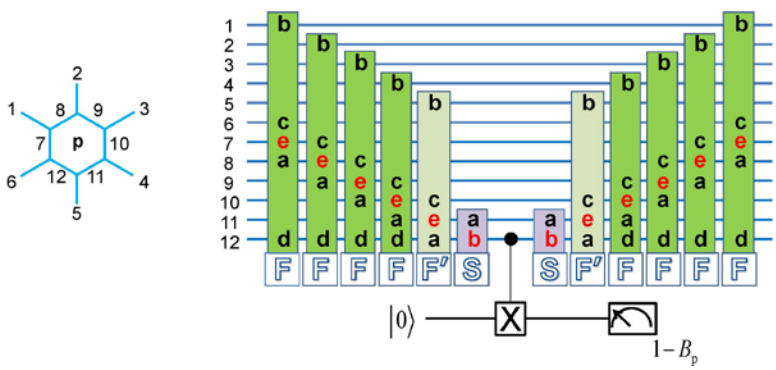

Figure 1. Quantum circuit for measuring the plaquette operator, B_p, for a hexagonal plaquette in the "Fibonacci" Levin-Wen model. The boxes represent quantum circuits which carry out so-called F- and S-moves (for precise descriptions of the quantum circuits inside each box, see [6]). The total gate count for this twelve-qubit quantum circuit is 8 five-qubit Toffoli gates, 2 four-qubit Tofolli gates, 10 three-qubit Toffoli gates, 43 CNOT gates and 24 single qubit gates.

Acknowledgements

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References

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