An Empirical Expression to Predict the Resonant Frequencies of Archimedean Spirals

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Introduction

Planar spiral resonators are used in a variety of applications including but not limited to planar inductors, wireless power transfer, meta-materials, filters, and nuclear magnetic resonance (NMR) probe coils. While the fundamental mode of the spiral is useful in NMR probe coils, the higher-order modes of a coil can degrade probe performance if they occur too close to other channels in the probe. Therefore, accurate determination of the resonant mode frequencies is an extremely important part of the design of resonant superconducting coils for NMR [1]. The goal of this study was to create a simple empirical formula capable of accurately predicting the resonance frequencies of Archimedean spiral resonators.

Experimental

A widely varying collection of spiral resonators were simulated in order to determine their resonant behavior. This collection included 81 spirals with different pitch, wire thickness, turn spacing, and ratio of inner and outer radii. The Hyperlynx software package from Mentor Graphics was used to conduct the simulations. Figure 1 shows the simulated reflection parameter of an inductively coupled spiral. These data were then analyzed to determine how the different characteristics of each spiral affected the resonant frequencies of the modes.

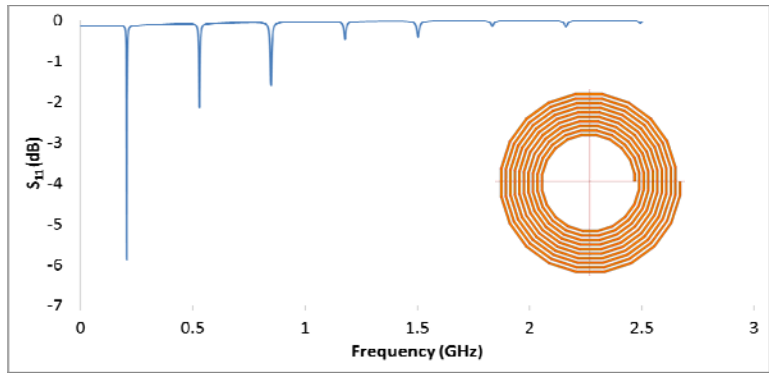

Fig.1: The resonances of a 10-turn spiral are shown by the reflection spectrum above. The actual spiral is also included in the lower-right corner.

Results and Discussion

The simulated resonance frequencies were then used to create the following equations which can be used to predict the resonance frequencies of freestanding Archimedean spirals.

$$f_n(Hz) = \left(\frac{v}{2L}\right)\left(0.24N^{-0.46} + 0.95\right)(n + D) \quad (1)$$

$$D = -\left((0.29 + 0.043F)\left(\frac{r_i}{r_o}\right) + 0.22\right)\left((F - 0.5)\left(2.65\left(\frac{P}{r_o}\right) + 0.03\right) + 1\right) \quad (2)$$

where f_n is the resonant frequency of the nth mode, F is the filling factor, P is the spiral pitch, n is the mode number, N is the number of turns, L is the length of the spiral conductor, and v is the velocity of EM waves in the surrounding medium. In this context filling factor refers to the ratio of the wire width to the spiral pitch. In addition to these empirical expressions, insights were obtained into the relationship between spiral parameters and the resonance frequencies. It was determined that the spiral spectrum is strongly affected by the ratio of the inner to outer radius of the spiral, and to a lesser extent the pitch and filling factor of the spiral.

The accuracy of the empirical expression (1) was found to be very good. For a total of 486 resonances that were simulated, the median error over the data set was only 0.87%, and more than 93% of all data points exhibited less than 2% error. This method is more easily applied and has a greater level of accuracy than the recently published method described in [2].

Conclusions

The presented formula is accurate and easily applicable to a diverse class of Archimedean spirals.

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References

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