**Effects of Dielectric Substrates on Resonance Frequency of Archimedean Spirals**

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**Introduction**

Superconducting self-resonant spiral structures are of current interest for applications both in metamaterials and as probe coils for nuclear magnetic resonance (NMR) spectroscopy [1]. Accurate spiral models are available in the literature for behavior of a spiral below and up to self-resonance. However, knowledge of the higher modes is also important. We present an empirical formula to calculate the multiple mode frequencies of single sided spirals on dielectric substrates based on the spiral parameters. In previous work, we studied Archimedean spirals in the absence of a dielectric substrate [2]. The basic approach of this work was to use an efficient simulation process to evaluate the resonance frequencies of a collection of spirals and use the resulting data to determine relationships between the spiral parameters and the resonances. In this work, we extend this approach to spirals backed by a dielectric substrate of finite thickness.

**Experimental**

When a spiral is backed by a dielectric substrate, the resonance frequencies are lower than the freestanding spiral. When the substrate is thick relative to the trace width of the spiral, the effect can be approximated by an effective dielectric constant [3]. However, this approximation is no longer valid when the substrate is thin relative to the trace width. The figure at right shows the effect of a thin substrate on the first four resonance frequencies of an example spiral. The frequencies, scaled as a fraction of the resonance frequency of the freestanding spiral, decrease at different rates for the different modes.

Simulation of the effect of substrate thickness as a fraction of trace width on the modes of an example spiral. The mode frequencies are normalized to the corresponding mode frequency in free space.

**Results and Discussion**

We propose that the mode frequencies of Archimedean spirals on dielectric substrates can be calculated by the following empirical expression.

[1]

[2]

[3]

[4]

[5]  
  
where, the spiral parameters *ri* (inner radius) and *ro* (outer radius), number of turns *N*, filling factor *F*, trace width *T*= *ro*- *ri*, and pitch *P*, and the dielectric constant and thickness *t* of the dielectric substrate are known. *v* is the velocity of electromagnetic waves in the surrounding medium, *n* is the mode number and *L* is the length of the spiral.

The formula proposed here is effective in modeling the resonance frequencies of planar Archimedean spirals on substrates with less than 2% error over a large range of substrate thicknesses. Many practical applications including superconductive NMR detectors utilize spirals on dielectric substrates, and the formula will be helpful for calculating the spectra of these devices.

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**References**

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