Evidence of two-stage melting of Wigner solids

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Ultralow carrier concentrations of two-dimensional holes down to $p = 1 \times 10^9$ cm^{-2} are realized. Remarkable insulating states are found below a critical density of $p_c = 4 \times 10^9$ cm^{-2} or $r_s \approx 40$. Sensitive dc *V-I* measurement as a function of temperature and electric field reveals a two-stage phase transition supporting the melting of a Wigner solid as a two-stage first-order transition.

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A Wigner crystal (WC) [1] of electrons in two dimensions is a long-sought-after phenomenon driven by strong electron-electron interaction and the melting of a WC provides a unique opportunity of understanding the solid-liquid transition (SLT) [2–4]. According to the Monte Carlo calculations, a WC occurs when the ratio of the interparticle Coulomb energy E_{ee} and the Fermi energy E_F, $r_s = E_{ee}/E_F = a/a_B$, is at least 37 [5]. $a = 1/\sqrt{\pi n}$ is the Wigner-Seitz radius for electron density n and $a_B = \hbar^2\epsilon/m^*e^2$ is the Bohr radius. Therefore, in order to observe a WC, the charge concentrations must be extremely dilute, i.e., $\leqslant 1 \times 10^9$ cm^{-2} for electrons or $\leqslant 4 \times 10^9$ cm^{-2} for holes in GaAs two-dimensional (2D) systems. Experiments in such small electron energy limits are challenging because the disorder effects, unless effectively suppressed, easily overwhelm the interaction-driven effects. Natural consequences are the Anderson localization [6], glass states [7], and mixed phases; all of which do not possess true long-range correlations. As a result, neither a WC nor a melting transition has been clearly demonstrated. Most detection efforts target collective modes and have so far produced only softly pinned modes undergoing a second-order-like thermal melting. These modes, as broadly suspected, could easily result from intermediate or mixed phases (e.g., hexatics, bubbles/stripes, or glass phases) since the observed correlation lengths (ξ), corresponding to the sizes of WCs, are usually small. Clear evidence of a WC demands not only demonstrations of longer or even macroscopic ξ, but, moreover, a melting transition marked by a singularity [8–10]. This work presents evidence for collective pinning modes characterized by a macroscopic ξ, as well as two-stage SLT, analogous to the Kosterlitz-Thouless (KT) model [4,10–15], except for a first-order transition suggested by a discontinuity across the critical point.

Most WC studies adopt the reentrant and quantum Hall insulating phases (RIP and QHIP) in a large magnetic field (B) where interaction effect is enhanced without reaching an ultradilute limit. Detection of the collective modes has been conducted with respect to pinning [16–18] and resonant absorption (via rf, microwaves, acoustic waves [19–22], tunneling [23]). However, there lacks evidence distinguishing a WC from intermediate/mixed phases. In fact, the estimated ξ is not only small (up to 1 μm), but also decays exponentially with increasing temperature (T) in a fashion similar to what is expected for an intermediate phase (i.e., hexatics [13,24]). Similar results are also obtained through studies in zero-B fields [25–27]. Zero-B results at $r_s > 40$ are quite rare due to the requirement for far more dilute carrier densities where disorder effect is even more prominent because the screening effect is weak as the interparticle spacing, $a = 1/\sqrt{\pi n} \sim$ 200–500 nm, approaches the screening length. This is why even fairly clean systems become highly insulating when n is $\sim$8–9 $\times 10^9$ cm^{-2} [27,28]. Consequently, r_s is limited to 5–15. Therefore, disorder suppression, as supported by almost all experiments, remains the key to successful detection of WCs.

In addition to driving a localization, another subtle disorder effect is its influence on the WC melting temperature (T_m), i.e. via fluctuations that break long-range translational symmetry. This less understood effect could alter current models of melting such as the KT model and is expected to be more effective in suppressing T_m than the quantum fluctuations [9,10]. Disorder suppression is therefore key in keeping T_m accessible. Most reported $T_m \sim$ 100–200 mK for small ξ cases are likely the crossover points between the intermediate/mixed and the liquid phases. To probe a transition, i.e., from a WC-intermediate phase, requires cooling to lower T.

This study focuses on the dc transport response of collective pinning and melting in ultrahigh-quality dilute 2D systems. A proven cooling method using a helium-3 immersion cell is adopted [29]. Key observations include enormously pinned collective modes, characterized by a differential resistance (r_d) of $\sim$1.3 GΩ, that exhibit a remarkable threshold nonlinear dc *I-V* identical to pinned charge density waves (CDWs). The

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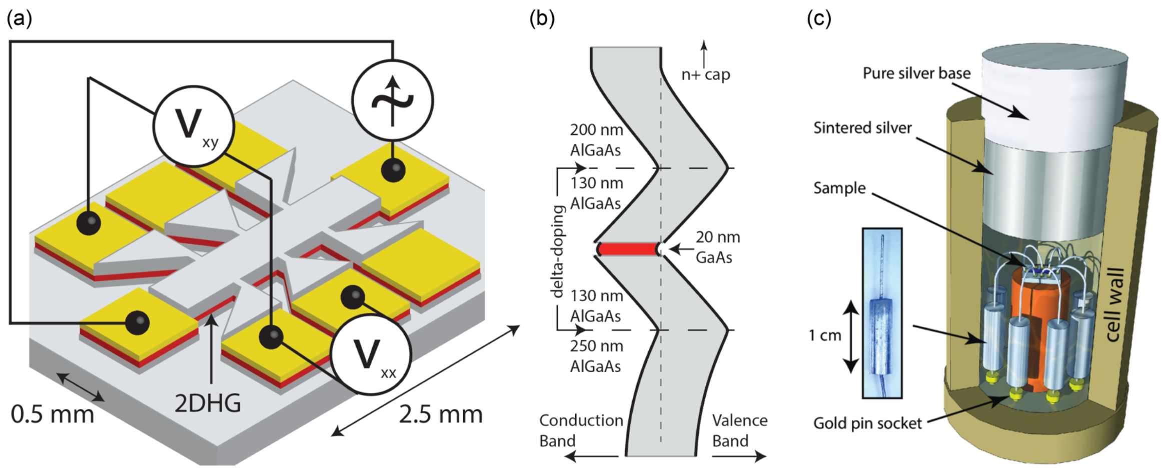

FIG. 1. (a) Sample dimensions and measurement configuration. (b) Band diagram of the quantum square well. (c) Cooling schematics inside a helium-3 immersion cell.

critical temperature is $T_m \sim 35$ mK. Moreover, heating across T_m results in a discontinuity in r_d which supports a first-order two-stage thermal melting. The presentation is divided into two parts: The first is a study of the RIP near filling $\nu = 1/3$ using p-doped quantum wells, and the second is a zero-B-field study of ultradilute holes in undoped heterojunction-insulated-gate field-effect transistors (HIGFETs) at $r_s \geqslant 40$.

The samples used for the RIP measurement are lightly doped p-type (100) GaAs quantum wells patterned into a 2.5×0.5 mm Hall bar. The density p is $\sim 4 \times 10^{10}$ cm^{-2} ($r_s = 24$), with mobility of $\mu \approx 2.5 \times 10^6$ cm^2/V s. Thermally deposited AuBe pads annealed at 460 °C achieve excellent Ohmic contacts to the 2D carriers, with measured contact resistances ~400 Ω. Measurements are performed in a dilution refrigerator inside a shielded room, allowing minimal electronic noises.

Cooling dilute carriers to 10 mK is challenging because sample thermalization relies mainly on cooling through the sample leads (via *ee* interaction) because the phonon modes are frozen out. We have established an effective cooling method via a helium-3 sample immersion cell and achieved 5 mK cooling GaAs 2D holes as dilute as $p = 5 \times 10^9$ cm^{-2} [29]. The carrier density for the RIP study here is nearly ten times higher. The cell is mounted at the lower end of a cold finger with its top fastened to the mixing chamber (mc) plate [Fig. 1(c)]. The roof is a sintered silver cylindrical-block extension made by compressed pure silver microparticles. During operation, helium-3 gas is continuously fed through a capillary into the cell where it condenses to fill the volume completely. Saturated sintered silver block provides ~30 m^2 contact area to cool the helium-3 bath. Major cooling of the 2D holes is realized via efficiently heat-sinking the metal contacts through sintered silver pillars providing 2.5 m^2 surface area per lead. T is monitored through a helium-3 melting curve thermometer. The T differential between the bath and the mc is $\leqslant 0.1$ mK at all times.

Figure 2(a) shows the magnetoresistance (MR) (ρ_{xx}) and the Hall resistance (ρ_{xy}) measured at 10 mK via a four-terminal ac technique. The inset shows the Shubnikov de Haas (SdH) oscillations starting at 0.05 T. The RIP peak centers at $B = 4.5$ T ($\nu = 0.375$) between fillings $\nu = 2/5$ and $1/3$, with a dip in ρ_{xy} consistent with previous studies [30,31]. $B = 4.5$ T corresponds to a magnetic length $l_B = \sqrt{hc/eB} \approx 28$ nm equal to $a = 1/\sqrt{\pi p}$. This is where a WC is expected. However, we found that dc techniques, instead of the ac techniques, are the appropriate method probing the RIP peak because, as shown later, it is essential to measured bulk resistance r with currents of ~1 pA which are far below the offset limits in the ac driving signals. An electrometer-level dc setup is therefore adopted with a voltage bias V between ±10 mV (at 0.1 μV resolutions). Current sensing via a low-noise preamp provides 50 fA precision.

Cooling to 9 mK, dc IV within a ±5 nA window displays a sharp threshold [inset of Fig. 2(b)] apparently identical to a pinned CDW [32]. The differential resistance $r_d = dV/dI$ within the threshold $V_c \sim 1$ meV is approximately 1.3 GΩ, with nearly no current flow ($I \leqslant$ 1–2 pA). This supports a collective pinning below a threshold electric field $E_c = V_c/L \sim 10$ mV/cm because the single-particle energy $w_c = eE_c a \sim 0.024$ μeV (or 0.3 mK) is significantly smaller than T. $L \sim 0.5$ mm is the distance between the voltage leads and $a = 1/\sqrt{\pi p} = 28$ nm is the average charge spacing. However, current is switched on immensely at a critical current I_c and r_d plummets by nearly 6000 times. It indicates a phase transition occurring at a remarkably small threshold current $I_c \sim 2$ pA. This electric field (E)-driven phase transition becomes more evident in the later T-dependent results. Joule heating is $< 10^{-15}$ W and thus negligible.

Another important evidence that supports a crystal phase is a melting transition which several studies have reported around 150–300 mK [16–23,25–27]. Figure 2(b) shows IVs with each of the curves corresponding to a fixed T between 10 and 300 mK. The threshold behavior is robust up to ~40 mK, with $r_d \sim$ 1–1.3 GΩ and $I_c \sim$ 2–3 pA. For higher T, the threshold behavior is replaced with rounded nonlinear IVs between 40 and 140 mK with substantially suppressed $r_d \sim$ MΩ. Eventually, linear IV is restored beyond 140 mK which is commonly recognized as a liquid phase due to the absence of pinning.

Naturally, a phase transition can be driven by both T and I (or V). Therefore, caution must be taken when examining the thermal melting because the sheet resistance $r = V/I$ are

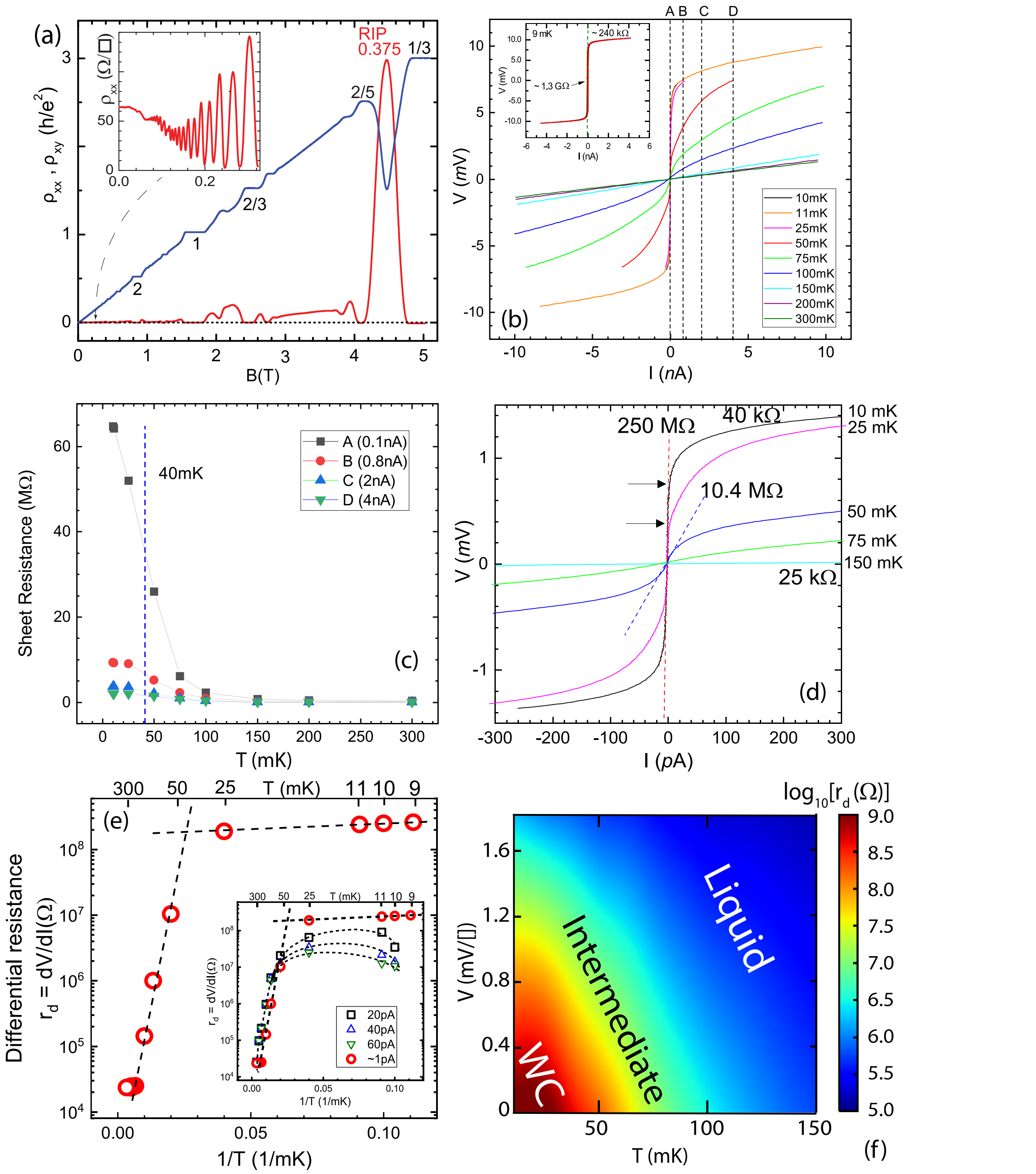

FIG. 2. (a) MR and Hall resistance at 10 mK. Inset: SdH oscillations. (b) dc *IV* measured at $B = 4.5$ T at various T from 10 to 300 mK. Inset: *IV* at 9 mK. (c) T dependence of the resistance (V/I) measured with 0.1, 0.8, 2, and 4 nA driving currents. (d) Amplified view of (b) for a narrower current range. (e) Piecewise T dependence of $r_d(T) = dV/dI|_{V\to 0}$ on semilogarithmic scales. Inset: comparison to $r_d(T)$ obtained with higher current drives. Dotted lines are guides. (f) Suggested contour phase diagram based on $\log_{10} r_d(T)$ values.

extremely sensitive to the level of the drives down to picoampere limits. For a demonstration, $r(T)$s obtained at four different randomly picked current drives, labeled by the dotted lines in Fig. 2(b), are plotted in Fig. 2(c). For $I \geqslant 1$ nA, $r(T)$ varies little with increasing I, consistent with a liquid phase behavior. However, $r(T)$ exhibits more than two orders increase already with $I \sim 100$ pA which must be linked to a transition effect.

Therefore, thermal melting must be examined in the limit of $I \to 0$. Figure 2(e) shows $r_d(T)|_{I\to 0}$ in comparison to the $r_d(T)$ obtained at 20, 40, and 60 pA. $r_d(T)|_{I\to 0}$ is well described as a piecewise behavior across a critical temperature of $\sim$35 mK defined as T_m. r_d decreases with increasing T at a rate of 1.5 MΩ/mK for $T \leqslant T_m$. For $T \geqslant T_m$, r_d exhibits an exponential dive marked by nearly four orders of magnitude down to 150 mK. The abrupt change within a few millikelvins of T_m is referred to as a discontinuity that supports, also confirmed by the zero-field results shown later, a possible first-order phase transition. The piecewise $r_d(T)$, however, disappears with I increased just beyond I_c. As shown in the inset of Fig. 2(e), $r_d(T)$ measured between

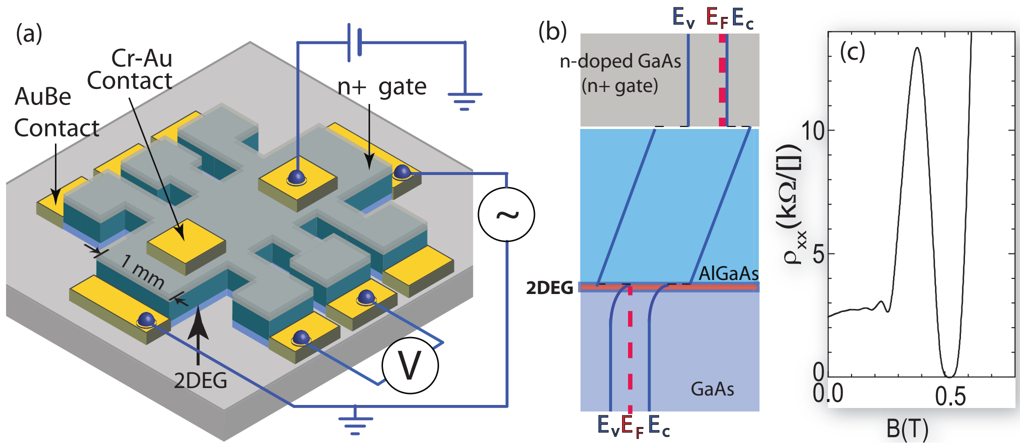

FIG. 3. (a) HIGFET sample and measurement schematics. (b) Band diagram showing accumulation of holes at the GaAs/AlGaAs junction. (c) $\rho_{xx}(B)$ for $p = 1.2 \times 10^{10}\ \mathrm{cm}^{-2}$.

20 and 60 pA is substantially suppressed at $T \leqslant T_m$ and only smooth nonmonotonic crossovers are found. These results are consistent with what is indicated by the threshold behavior that a phase transition has occurred when $I > I_c$.

Meanwhile, although pinning at $T < T_m$ is consistently strong, V_c exhibits a noticeable T dependence. Figure 2(d) shows selected IVs for 10, 25, 50, 75, and 150 mK within a narrower window. Note that r_d is now shown as resistivity (instead of resistance). For 10 and 25 mK, current switches on at different thresholds: 0.4 mV for 25 mK and 0.8 mV for 10 mK. Lower E_c for higher T is qualitatively consistent with the Lindemann criterion for crystal melting [14]. An estimate of ξ is provided here, similar to previous studies [18,33], based on a pinning model [34,35] that balances the pinning energy with the electrical potential energy $U = N w_c$. $w_c = eE_c a \sim 0.024\ \mu\mathrm{eV} \ll T$ is the single-particle potential energy. $N = p\xi^2$ is the number of carriers on a scale of ξ. For $T = 25$ mK, $E_c = V_c/L \sim 8$ mV/cm where $L = 0.5$ mm. Setting the electrical force NeE_c equal to the pinning force κa [17,18], κ being the shear modulus $0.245 e^2 p^{3/2}/4\pi\epsilon_0\epsilon$ [36], $N \sim 1.5 \times 10^5$ or $\xi \geqslant 10\ \mu$m is obtained. U is ~2.4 meV (or 30 K), comparable to E_{ee}.

For T between 40 and 140 mK, the threshold is replaced with a rounded nonlinear IV. This observation is in agreement with several previous results [17,25–27] which were interpreted as pinned WCs. However, E_c disappears because a current switches on in the limit of $V \to 0$. In addition, pinning is substantially reduced, i.e., $r_d \sim 10$ MΩ at 50 mK. Therefore, this region should be of an intermediate phase since it crosses over to a liquid above 140 mK (referred to as T_l). r_d for $T \geqslant T_l$ is 30–40 kΩ/□. The same values of r_d are found for the liquid phase arrived at by E-field-driven melting at sufficiently large bias. A suggested phase diagram is shown in Fig. 2(f).

To identify the nature of the intermediate phase is difficult because the exact relationship between r_d and $\xi(T)$ has to first be formulated. Here, as a minor point, we show that $r_d(T)$ can be fitted to $r_d = r_0 \exp[c/(T - 40\ \mathrm{mK})^\gamma]$ with $r_0 \approx 23$

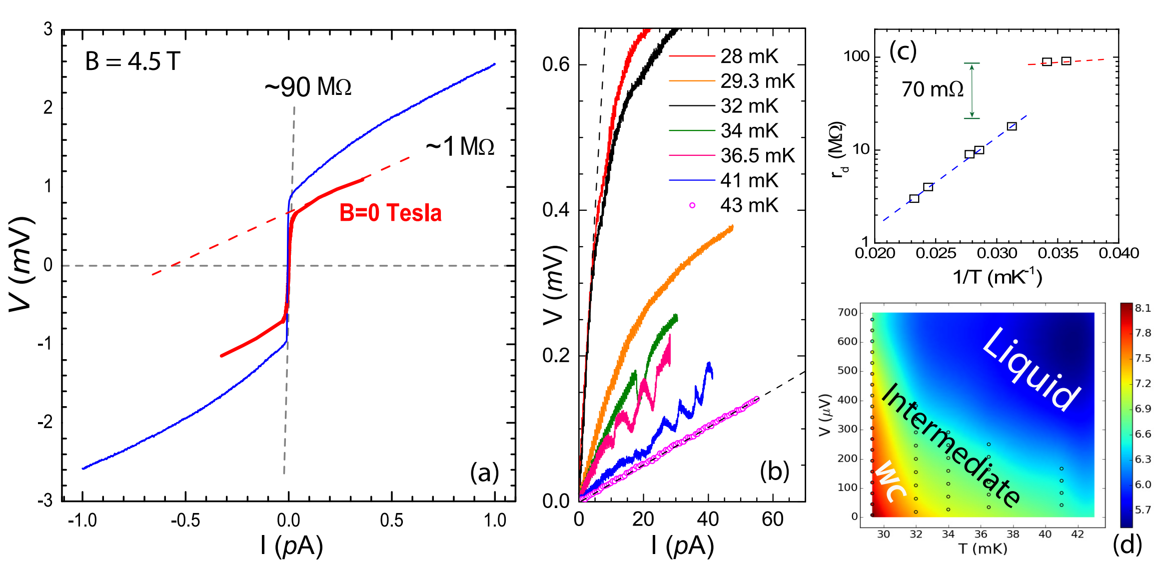

FIG. 4. (a) dc IV at $T = 28$ mK. (b) IVs obtained at different T. (c) T dependence of $r_d(T)|_{V\to 0}$ on semilogarithmic scales. (d) Colored contour phase diagram based on $\log_{10} r_d(T)$ values. Dashed lines are guides.

and $c \approx 9.5$, in the same trend as the exponentially decreasing $\xi(T)$ modeled for a hexatic phase [13]: $\xi \sim \exp[c/(T - T_m)^\gamma]$ ($\gamma \approx 0.3696$).

We now turn to the zero-B-field study with undoped GaAs/AlGaAs HIGFETs [37–39]. A 6 mm × 0.8 mm Hall bar is realized with a self-align fabrication process [39] [Fig. 3(a)]. Accumulation of holes at the heterointerface is capacitively induced through biasing a top gate beyond a turn-on voltage, ~ -1.3 V, at which the valance band edge meets the chemical potential [Fig. 3(b)]. The band gap of the 600-nm-thick $Al_{0.3}Ga_{0.7}As$ barrier is ~2 eV. Owing to the superior crystal quality, gate leakage remains less than 0.05 pA at all operating bias. Density p, determined via quantum Hall oscillations [Fig. 3(c)], is tunable from 4×10^{10} down to 7×10^{8} cm^{-2}.

Accessing $r_s \geqslant 40$ requires $p \leqslant 4 \times 10^9$ cm^{-2}. $m^* = 0.25 m_0$ is a lower-bound estimate. (Determination of m^* is difficult because of a complicated dispersion relation associated with the light-heavy hole band mixing and the spin-orbit coupling [40].) It is thus important to exclude disorder-driven localization which easily occurs as phonon-activated hopping $\rho(T) \sim \rho_0 \exp{(T^*/T)^{1/\beta}}$ ($\beta = 1$–3) [41,42]. Recent studies of ultraclean systems revealed nonactivated power-law behaviors [38,43,44] of interaction-driven nature that distinguishes from a disorder-driven effect. And, the metal-to-insulator transition (MIT) [45] occurs at lowest carrier densities corresponding to $r_s \sim 35$–40. We refer the readers to Refs. [38,43,44,46] for details.

The measured MIT in zero B (not shown) has a critical density $p_c = 4 \times 10^9$ cm^{-2}. The following dc results are for $p = 2.8 \times 10^9$ cm^{-2} (or $r_s \sim 45$) measured between 28 and 45 mK. A current bias, with Keithley 6430 fA source, is employed with a voltage sensing at sub-μV resolution at an input impedance of 10^{16} Ω. Figure 4(a) shows a similar threshold IV obtained at 28 mK, qualitatively identical to the RIP case. I_c is ~4 pA. Strong subthreshold pinning is marked by a r_d of 90 MΩ/□. The suprathreshold r_d collapses 100 times. I_c corresponds to a $E_c \sim 4$ mV/cm (or $\sim 10^{-10}$ V/a_B), yielding a slightly larger single-particle potential energy of $\sim eE_c a \sim 0.04$ μeV (or 0.46 mK) due to the larger $a \sim 100$ nm. Setting $NeE_c = \kappa a$ as shown earlier, one obtains $N \sim 1 \times 10^5$, corresponding to a substantial scale of $\xi \sim 100$ μm. This yields a dominating potential energy $U \sim 20$ meV $> E_{ee}$ which is consistent with a crystal. For a consistency check, the same setup is used to measure the RIP and the result is shown as the blue curve. The power dissipation is $\leqslant 2 \times 10^{-16}$ W, ruling out appreciable Joule heating.

Melting probed by $r_d|_{I\to 0}$ is shown in Fig. 4(c) where a piecewise behavior appears across $T_m \sim 30$ mK. dr_d/dT is 4 MΩ/mK for $T < T_m$. r_d exhibits a sharp jump of 70 MΩ at T_m above which an exponential T dependence is found. E_c disappears at $T > T_m$ where rounded nonlinear IV is found. The discontinuous jump resembles a recent quantum Monte Carlo simulation for a first-order WC-intermediate phase transition mediated by a discontinuous internal energy jump [10], and supports a singularity dividing a WC from an intermediate phase. Linear IV is recovered at $T_l \sim 42$ mK, noticeably lower than the RIP case. Smaller T_m and T_l for the ultradilute case is qualitatively consistent with stronger quantum fluctuations and disorder fluctuations (due to lack of screening). A phase diagram is suggested in Fig. 4(d).

Increasing E-field results in a switch-on of current and a settlement of r_d belonging to a liquid phase. Identical to the RIP case, a melting mediated by an intermediate phase is supported. However, there is a noticeable difference in the intermediate phase at $T > T_m$: V oscillates with increasing I at approximately 5–10 pA spacing. It occurs more frequently as T approaches T_l [Fig. 4(b)]. The formation of stripes with long-range orientational order [13], as seen in electrons on a helium surface [47], could be a possible cause. Another possibility is that small T_l facilitates a melting and recrystallization of pinned WC domains, instead of or in addition to shearing, when driven across pinning sites [48]. This will contribute to a negative r_d.

To summarize, enormous pinning modes below T_m support a WC on large ξ scales. A melting is captured as a two-stage SLT. The WC-intermediate phase transition is likely first-ordered [10] because of the discontinuity in r_d as well as the disappearance of E_c above T_m. Results obtained from both RIP and zero-B-field studies are remarkably consistent. The small T_m, which is $\sim(1/7)T_{cm}$, suggests strong effects from system disorders and quantum fluctuations that require further understanding.

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