From Birefringent Electrons to a Marginal or Non-Fermi Liquid of Relativistic Spin-1/2 Fermions: An Emergent Superuniversality

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We present the quantum critical theory of an interacting nodal Fermi liquid of quasirelativistic pseudospin-3/2 fermions that have a noninteracting birefringent spectrum with two distinct Fermi velocities. When such quasiparticles interact with gapless bosonic degrees of freedom that mediate either the long-range Coulomb interaction or its short range component (responsible for spontaneous symmetry breaking), in the deep infrared or quantum critical regime in two dimensions, the system is, respectively, described by a marginal- or a non-Fermi liquid of relativistic spin-1/2 fermions (possessing a unique velocity), and is always a marginal Fermi liquid in three dimensions. We consider a possible generalization of these scenarios to fermions with an arbitrary half-odd-integer spin, and conjecture that critical spin-1/2 excitations represent a superuniversal description of the entire family of interacting quasirelativistic fermions.

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Introduction.—All fermions in the standard model have spin 1/2; however, higher spin particles, such as the gravitino, a charge-neutral spin-3/2 fermion, have been postulated in theories such as supergravity [1]. An important recent advance in condensed matter physics is the discovery of (quasi)-relativistic spin-1/2 fermions in graphene [2], on the surface of topological insulators [3–5], in Weyl materials [6], and in topological superconductors [7]. It is also conceivable to realize higher spin fermions as emergent quasiparticles in various solid state systems in the vicinity of band-touching points [8–20], which can be either symmetry protected or correspond to a fixed point description of a quantum phase transition between two topologically distinct insulators.

Pseudo-spin-3/2 fermions [21] can be found in the close proximity of linear or biquadratic touching of valence and conduction bands [8]. We focus on the former situation where the quasiparticles display a birefringent spectrum with two distinct Fermi velocities and, therefore, manifestly break Lorentz symmetry. Such fermions can be realized from simple tight-binding models on a two-dimensional generalized π-flux square lattice [9–11], honeycomb lattices [12,13], shaken optical lattices [14,15], as well as in three-dimensional strong spin-orbit coupled systems [16,17], such as antiperovskites [18] and the CaAgBi family of materials [19]. In the present Letter, we venture into the largely unexplored territory [11,12,20] that encompasses the response of such peculiar gapless fermionic excitations and their stability in the presence of electronic interactions.

Now, we provide a brief summary of our main findings. Irrespective of their materials origin and dimensionality of the system, we show that the optical conductivity of noninteracting spin-3/2 fermions at zero temperature is identical to that of pseudorelativistic spin-1/2 fermions. When spin-3/2 fermions interact with massless bosonic degrees of freedom (d.o.f.), which mediate either the long-range Coulomb interaction or its short-range component, in the deep infrared or quantum critical regime, a marginal Fermi liquid of effective spin-1/2 fermions emerges, featuring logarithmic corrections to its Fermi velocity in three dimensions. By contrast, in two spatial dimensions, the system respectively hosts a marginal Fermi liquid or a non-Fermi liquid of relativistic spin-1/2 fermions. At the non-Fermi liquid fixed point, the residue of the quasiparticle pole vanishes in a power-law fashion, and the ordered phase for a strong repulsive interaction represents a quadrupolar charge- or spin-density wave, while it is a spin-singlet s-wave paired state in the case of a strong attractive interaction. Finally, based on the form of the Hamiltonian for arbitrary half-odd-integer spin relativistic fermions, we conjecture that the corresponding nodal liquid is ultimately described in terms of critical spin-1/2 fermions, which promotes these excitations as the superuniversal description of the entire family of interacting quasirelativistic fermions.

Hamiltonian.—The low-energy Hamiltonian describing a collection of quasirelativistic pseudospin-3/2 fermions is given by $\tilde{\eta} \otimes H_{3/2}(\mathbf{k})$, where

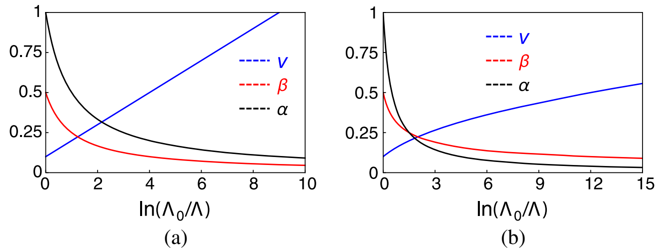

FIG. 1. Renormalization group (RG) flow of the mean Fermi velocity (v), birefringence parameter (β), and fine structure constant (α) for quasirelativistic pseudospin-3/2 fermions in the presence of an instantaneous long-range Coulomb interaction in (a) $d=2$ and (b) $d=3$. The bare values of the parameters are chosen to be $v_0=0.1$, $\beta_0=0.5$, $\alpha_0=1$.

$$H_{3/2}(\mathbf{k}) = v\sum_{j=1}^{d}[\Gamma_{j0}k_j + \beta\Gamma_{0j}k_j], \tag{1}$$

in dimensions $d=2$ and $d=3$ with $\tilde{\eta}$ defined below. The four component Hermitian matrices are $\Gamma_{\mu\nu}=\sigma_\mu\otimes\sigma_\nu$, where $\{\sigma_\mu\}$, $\mu=0,1,2,3$ are the standard two dimensional Pauli matrices, and $\mathbf{k}=(k_1,\ldots,k_d)$. The isotropic spectrum of $H_{3/2}(\mathbf{k})$ is given by $\pm v(1\pm\beta)|\mathbf{k}|=\pm v_\pm|\mathbf{k}|$, displaying birefringence with two effective Fermi velocities $v_\pm = v(1\pm\beta)$, where β is the birefringence parameter. Notice the spectra of $\tilde{H}_{3/2}(\mathbf{k})=v(\mathbf{S}\cdot\mathbf{k})$, namely, $\pm v(3,1)|\mathbf{k}|/2$, where $\mathbf{S}$ are spin-3/2 matrices and are recovered for $\beta=1/2$. For $\beta=0$ we recover two decoupled copies of quasirelativistic pseudospin-1/2 fermions, similar to the situation in monolayer graphene [2] or the regular π-flux square lattice [22]. For $\beta=1$, the spectrum accommodates two flat bands and a Dirac cone, as found for the Lieb lattice [23]. The Hamiltonian in Eq. (1) permits the most general birefringent structure. Here, we restrict ourselves to $|\beta|<1$. In $d=2$, the Hamiltonian in Eq. (1) describes low-energy excitations in a generalized π-flux square lattice, with $\tilde{\eta}\equiv\eta_0$ [9–11], while Pauli matrices $\{\eta_\mu\}$ act on the spin indices. In contrast, in three dimensions, $H_{3/2}(\mathbf{k})$ could describe spin-3/2 Weyl excitations in a system with strong spin-orbit coupling, with $\tilde{\eta}\equiv\eta_3$, and the $\{\eta_\mu\}$ operating on valley indices. Independent of these microscopic details, the minimal representation of quasirelativistic spin-3/2 fermions is four dimensional, in contrast to spin-1/2 fermions for which the minimal representation is two dimensional.

Optical conductivity.—First, we focus on the response of spin-3/2 fermions to an external electromagnetic field and compute the optical conductivity at temperature $T=0$. The current operator in the lth direction is $j_l = v(\Gamma_{0l}+\beta\Gamma_{l0})$, where $l=1,\ldots,d$. To extract the optical conductivity in a d-dimensional noninteracting system, we compute the polarization bubble [24]

$$\Pi^{(d)}(i\Omega) = -\frac{e_0^2}{\hbar d}\sum_{l=1}^{d}[\Pi_{ll}(i\Omega)-\Pi_{ll}(i\Omega=0)], \tag{2}$$

where e_0 is the external electronic test charge and

$$\Pi_{ll}(i\Omega) = \mathrm{Tr}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}j_l G_{\mathbf{k}}(i\omega+i\Omega)j_l G_{\mathbf{k}}(i\omega). \tag{3}$$

Here, $G_{\mathbf{k}}(i\omega)$ is the fermionic Greens function in terms of the Matsubara frequency ($i\omega$). Then, we perform analytic continuation to real frequencies (ω) via $i\Omega\to\omega+i\delta$ and use the Kubo formula to obtain the optical conductivity

$$\sigma_{ll}^{(2)}(\omega)=\frac{e_0^2}{h}\frac{\pi N_f}{4}, \qquad \sigma_{ll}^{(3)}(\omega)=\frac{e_0^2}{h}\frac{N_f\omega}{6v}, \tag{4}$$

respectively, in $d=2$ and 3, where N_f is the number of four-component spin-3/2 fermion species. Note that the optical conductivity in a nodal Fermi liquid of spin-3/2 fermions with N_f flavors is identical to that of $2N_f$ component spin-1/2 quasirelativistic Dirac or Weyl fermions [2,6], since $\sigma_{ll}^{(d)}(\omega)$ does not depend on β. Therefore, the above example already indicates that the birefringence parameter (β) may not be important for the physical properties of this state at $T=0$, in the absence of an infrared cutoff. Now, we include the interactions to study their effects on spin-3/2 fermions, and show that their main role is to make the birefringence parameter irrelevant and restore Lorentz symmetry.

Long-range Coulomb interaction.—First, we focus on the long-range tail of the density-density Coulomb interaction, and neglect the retarded current-current interaction, since, in a condensed matter system, $v_\pm \ll c$ (the speed of light). The imaginary-time (τ) action capturing the instantaneous Coulomb interaction is

$$S_C = \int d\tau d^d\mathbf{r}d^d\mathbf{r}'\rho(\tau,\mathbf{r})V(\mathbf{r}-\mathbf{r}')\rho(\tau,\mathbf{r}'), \tag{5}$$

where $V(\mathbf{r}-\mathbf{r}')=e^2/|\mathbf{r}-\mathbf{r}'|$, and $\rho(\tau,\mathbf{r})$ is the fermionic density. In reciprocal space, the Coulomb interaction $V(\mathbf{k})\sim e^2/|\mathbf{k}|^{d-1}$ is an analytic (nonanalytic) function of momentum in three (two) dimensions. Therefore, only in $d=3$ is charge dynamically screened by fermions, since the fermion bubble can only yield corrections that are analytic in momentum [25–31]. From the leading order fermionic and bosonic (soft gauge field mediating Coulomb interaction) self-energy corrections, we arrive at the following renormalization group flow equations [24]:

$$\frac{dv}{d\ell}=\frac{\alpha v}{C_d}\equiv\frac{e^2}{C_d}, \qquad \frac{d\beta}{d\ell}=-\frac{\alpha\beta}{C_d},$$
$$\frac{d\alpha}{d\ell}=-(1+N_f\delta_{d,3})\frac{\alpha^2}{C_d}, \tag{6}$$

where $\alpha=e^2/v$ is the fine structure constant, $C_2=8\pi$, $C_3=6\pi^2$, and $\ell\equiv\ln(\Lambda_0/\Lambda)>1$ is the logarithm of the running renormalization group scale, with Λ_0 the ultraviolet cutoff, while Λ is the running scale. To leading order, the birefringent part of $H_{3/2}(\mathbf{k})$ remains marginal in the presence of Coulomb interactions, namely $d(\beta v)/d\ell = 0$, while the mean Fermi velocity (v) receives a β-independent logarithmic correction making v a marginally relevant parameter.

Thus, in a two or three dimensional interacting quasirelativistic liquid of spin-3/2 fermions, the mean Fermi velocity (v) increases logarithmically, while the birefringent parameter (β) decreases, also logarithmically, but the parameter βv remains marginal. Ultimately, in the deep infrared regime, we effectively recover two decoupled copies of relativistic spin-1/2 fermions, since $v_+ - v_- \ll v$ yielding $\beta v \ll v$. We dub such a state an effective marginal Fermi liquid of spin-1/2 fermions. As a result, the fine structure constant (α) also decreases logarithmically slowly. These scenarios for $d = 2$ and $d = 3$, respectively, are shown in Figs. 1(a) and 1(b).

Because of our neglect of the retarded part of the long range interaction, the enhancement of the mean Fermi velocity (v) is unbounded. However, as v increases, the current-current interaction (originally suppressed by $v_\pm/c \ll 1$) can no longer be neglected, and ultimately, the flow of the Fermi velocity stops at the speed of light (c), leading to the restoration of genuine Lorentz symmetry [29–31]. Although the parameter βv is initially marginal, based on the results with retarded short-range interaction mediated by a bosonic order-parameter field, which we discuss below, we suspect that, ultimately, βv [originally a marginal or scale (ℓ) invariant quantity in the presence of only instantaneous Coulomb interaction] becomes irrelevant with the inclusion of retarded long-range current-current interaction, implying $\beta v \to 0$ as $\ell \to \infty$ (see Fig. 2 for qualitative comparison). Therefore, at the lowest energy scales, once the full electromagnetic interaction is accounted for, only spin-1/2 fermions survive. However, the length scale (ℓ_*) at which the current-current interaction becomes important is extremely large ($\ell_* \gg 1$), and logarithmically slow growth of the Fermi velocity makes it impossible to access such a regime in any real system. We leave this issue of definite fundamental importance but pure academic interest for future investigation. In $d = 3$, besides the Lorentz symmetric fixed point, it is also conceivable to find an O_h (cubic) symmetric interacting fixed point in a crystalline environment. However, the Lorentz symmetric fixed point has a larger basin of attraction [32].

Proximity to a Mott transition.—Finally, we address the quantum critical description of a collection of spin-3/2 fermions, residing near a Mott transition, driven into an insulating phase by the short range parts of the Coulomb interaction (such as those appearing in an extended Hubbard model) [11,20]. The density of states vanishes as $\varrho(E) \sim |E|$; hence, any ordering takes place at a finite coupling through a quantum phase transition. For spinless

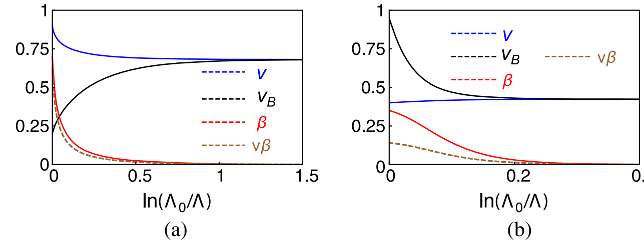

FIG. 2. RG flow of two Fermi velocities (v and $v\beta$), bosonic (v_B) velocity, and the birefringence parameter (β) in the presence of Yukawa interaction ($g^2 \sim \epsilon$) when (a) $v_0 > v_{B,0}$ and (b) $v_{B,0} > v_0$, for $N_f = N_b = 1$. For panel (a), we choose $v_0 = 0.9$, $v_{B,0} = 0.2$, $\beta_0 = 0.7$, $v_0\beta_0 = 0.63$ and for (b) $v_0 = 0.4$, $v_{B,0} = 0.95$, $\beta_0 = 0.35$, $v_0\beta_0 = 0.14$.

fermions in $d = 2$, there exists one matrix, namely, Γ_{33}, that anticommutes with $H_{3/2}(\mathbf{k})$. Thus, in an ordered phase with $\langle \Psi^\dagger \eta_0 \otimes \Gamma_{33} \Psi \rangle \neq 0$, the quasiparticle spectrum is fully and uniformly gapped. Hence, at $T = 0$, the propensity toward such an ordering is energetically favored since it maximally lowers the free energy. Such an ordered phase represents a quadrupolar charge-density wave, since $\Gamma_{33} = (2S_z^2 - S_x^2 - S_y^2)/3$, when $S_z = \mathrm{diag}(3, 1, -1, -3)/2$. The inclusion of spin d.o.f. leads to a competing order, also representing a massive ordered phase, where $\langle \Psi^\dagger \vec{\eta} \otimes \Gamma_{33} \Psi \rangle \neq 0$ describes a quadrupolar spin-density wave, with $\vec{\eta} = (\eta_1, \eta_2, \eta_3)$. Both spin- and charge-density-wave phases break the discrete particle-hole symmetry generated by Γ_{33}, as $\{H_{3/2}(\mathbf{k}), \Gamma_{33}\} = 0$, while the former one also breaks the continuous $SU(2)$ spin-rotational symmetry generated by $\vec{\eta} \otimes \Gamma_{00}$; hence, the ordered phase is accompanied by two Goldstone modes. These two phases can be realized for sufficiently strong on-site [20] and nearest-neighbor [11] repulsion, respectively, in a π-flux square lattice. By contrast, in $d = 3$ there is no matrix that anticommutes with $H_{3/2}(\mathbf{k})$ and single-flavored spin-3/2 Weyl fermions cannot be gapped out. Nevertheless, if we account for valley d.o.f., then spin-3/2 Weyl fermions can be gapped out by spontaneously breaking translational symmetry (generated by $\eta_3 \otimes \Gamma_{00}$), and the ordered phase is characterized by $\langle \Psi^\dagger \vec{\eta}_\perp \otimes \Gamma_{00} \Psi \rangle \neq 0$, with $\vec{\eta}_\perp = (\eta_1, \eta_2)$. The same order parameter can represent a spin-singlet s-wave pairing for spin-3/2 fermions in $d = 2$ (with an appropriate redefinition of the spinor basis). However, the fate of an emergent Lorentz symmetry close to a Mott transition for two or three-dimensional linearly dispersing spin-3/2 fermions is insensitive to these details.

The imaginary time action describing such a quantum phase transition is given by $\mathcal{S} = \int d\tau d^d\mathbf{r} (\mathcal{L}_f + \mathcal{L}_Y + \mathcal{L}_b)$, where $\mathcal{L}_f$ describes the dynamics of gapless spin-3/2 fermionic excitations (represented by a spinor field Ψ) with

$$\mathcal{L}_f = \Psi^\dagger [\partial_\tau + \tilde{\eta} \otimes H_{3/2}(\mathbf{k} \to -i\nabla)] \Psi, \qquad (7)$$

where $\tilde{\eta} = \eta_0$ or η_3. The coupling between gapless bosonic and fermionic d.o.f. is captured by

$$\mathcal{L}_Y = g\sum_{\alpha=1}^{N_b}\Phi_\alpha\Psi^\dagger M_\alpha\Psi, \tag{8}$$

where g is the Yukawa coupling, and N_b counts the number of real components of the bosonic order-parameter field. Therefore, $N_b = 1(3)$ for a quadrupolar charge(spin)-density wave in $d = 2$, and $N_b = 2$ for a translational symmetry breaking charge-density wave in $d = 3$ and s-wave pairing in $d = 2$. The Hermitian matrix M_α always anticommutes with the noninteracting Hamiltonian $\tilde{\eta} \otimes H_{3/2}(\mathbf{k})$. The dynamics of the order-parameter bosonic field is captured by an appropriate relativisticlike Φ^4 theory

$$L_b = \sum_{\alpha=1}^{N_b}\left[-\frac{1}{2}\Phi_\alpha\left(\partial_\tau^2 + v_B^2\sum_{j=1}^{d}\partial_\mu^2 - m_b^2\right)\Phi_\alpha + \frac{\lambda}{4!}[\Phi_\alpha^2]^2\right], \tag{9}$$

where λ is the four-boson coupling, m_b^2 (the bosonic mass) is the tuning parameter for the transition with $m_b^2 = 0$ at the quantum critical point, and v_B is the characteristic velocity of the bosonic field. Note that both Yukawa and four-boson couplings are dimensionless in $d = 3$. Therefore, the critical behavior of the above field theory can be addressed by performing a controlled ϵ expansion about three spatial dimensions, with $\epsilon = 3 - d$ [33]. From the leading order self-energy corrections, we arrive at the following flow equations for the two velocities (v and v_B) and the birefringence parameter (β) [24]:

$$\begin{aligned}\frac{dv}{d\ell} &= -2N_b v g^2 A(v, v_B, \beta),\\ \frac{d\beta}{d\ell} &= -2N_b\beta g^2 S(v, v_B, \beta),\\ \frac{dv_B}{d\ell} &= \frac{N_f g^2 v_B}{2v^3(1-\beta^2)}\left(C(v, v_B, \beta) - \frac{1}{1-\beta^2}\right),\end{aligned} \tag{10}$$

with $X(v, v_B, \beta) \equiv X$ for $X = A$, S, C and where

$$\begin{aligned}A &= \frac{2(v - v_B)(v + v_B)^2 + 4v^3\beta^2}{3vv_B[(v + v_B)^2 - v^2\beta^2]},\\ S &= \frac{2[4vv_B(v + v_B) + v_B^3 + v^3(1-\beta^2)]}{3vv_B[(v + v_B)^2 - v^2\beta^2]},\\ C &= \frac{v^2}{v_B^2} - \beta^2\left[\frac{4}{(1-\beta^2)^2} - \frac{2v^2}{3v_B^2}\right].\end{aligned} \tag{11}$$

Even though it is a daunting task to solve the above coupled flow equations exactly, valuable insights can be gained from their numerical solutions, displayed in Fig. 2.

We note that, regardless of whether $v > v_B$ [Fig. 2(a)] or $v_B > v$ [Fig. 2(b)] in the bare theory, the Fermi and bosonic velocities approach each other, while the birefringent parameter β flows to zero in the deep infrared regime, but in this case, the parameter βv also flows, separately, to zero. Hence, as the system approaches the boson-fermion coupled Yukawa fixed point, it is effectively described by two decoupled copies of spin-$1/2$ fermions and the Lorentz symmetry gets restored since v_+ and v_- approach a common velocity, v, $v_B \to \tilde{v}$, and $v\beta \to 0$ (see brown dashed line in Fig. 2) as $\ell \to \infty$. The coupled flow equations for the remaining two couplings in the $\beta = 0$ and $m_b^2 = 0$ hyperplane take the form

$$\frac{dg^2}{d\ell} = \epsilon g^2 - a_1 g^4, \qquad \frac{d\lambda}{d\ell} = \epsilon\lambda - 4N_f g^2[\lambda - 6g^2] - \frac{a_3\lambda^2}{6}, \tag{12}$$

and support only one quantum critical point located at

$$(g_*^2, \lambda_*) = \left(1, \frac{3}{a_3}[a_2 + \sqrt{a_2^2 + 16N_f a_3}]\right)\frac{\epsilon}{a_1}, \tag{13}$$

where $a_1 = 2N_f + 4 - N_b$, $a_2 = 4 - 2N_f - N_b$, and $a_3 = N_b + 8$. At this critical point, both fermionic and bosonic excitations possess nontrivial anomalous dimensions, given by $\eta_f = N_b g_*^2/2$ and $\eta_b = 2N_f g_*^2$, respectively, responsible for the absence of sharp quasiparticles in its vicinity. Specifically, the residue of the fermionic quasiparticle pole vanishes as $Z_\Psi \sim (m_f)^{\eta_\Psi/2}$, where m_f is the fermionic mass that vanishes following a universal ratio $(m_b/m_f)^2 \sim \lambda_*/g_*$ as the critical point is approached from the ordered side. Therefore, in $d = 2$ or $\epsilon = 1$, Z_Ψ vanishes in a power-law fashion, and the Yukawa critical point accommodates a relativistic non-Fermi liquid, where gapless spin-$1/2$ fermionic and bosonic order-parameter excitations are strongly coupled. By contrast, in $d = 3$ or $\epsilon = 0$, the critical phenomena at the transition are controlled by a Gaussian fixed point, located at $g_*^2 = \lambda_* = 0$, which exhibit mean-field behavior with logarithmic corrections due to the fact that the field theory is at its upper critical dimension. Consequently, the residue of the quasiparticle pole vanishes only logarithmically, and the Yukawa fixed point hosts a relativistic marginal Fermi liquid.

Discussion.—We demonstrate that, when quasirelativistic spin-$3/2$ fermions interact with bosonic d.o.f. which represent either a soft gauge field mediating the long-range or an order-parameter field mediating a short-range Coulomb interaction, in the deep infrared regime, the system is described by either an effective marginal- or non-Fermi liquid of relativistic spin-$1/2$ excitations [34]. One can envision generalizing our analysis to spin-$s/2$ quasirelativistic fermions with an arbitrary odd value of s. For spin-$s/2$ Weyl fermions, the low-energy Hamiltonian can be written as $\tilde{H}_{s/2}(\mathbf{k}) = v(\mathbf{S}\cdot\mathbf{k})$, where $\mathbf{S}$ are now spin-$s/2$ matrices, and the quasiparticle spectrum has $(s+1)/2$ effective Fermi velocities, given by $v_s = (1/2, 3/2, \ldots, s/2)v$. Even though we expect our conclusions to hold for any value of s, specifically when $s + 1 = 2^N$, with

N an integer, a more general multirefringence can be accommodated by the Hamiltonian

$$H_{s/2}(\mathbf{k}) = v \sum_{j=1}^{d} k_j [\Gamma_{j0\cdots0} + \cdots + \beta_{N-1}\Gamma_{0\cdots0j}], \quad (14)$$

similar to Eq. (1). Here, $\Gamma_{j0\cdots0} = \sigma_j \otimes \sigma_0 \otimes \cdots \otimes \sigma_0$ and so on are $(s+1)$-dimensional Hermitian matrices. Note that, for $d = 2$, there exists a matrix, namely $\Gamma_{33\cdots3} = \sigma_3 \otimes \cdots \otimes \sigma_3$, that fully anticommutes with $H_{s/2}(\mathbf{k})$. Therefore, we believe that our proposed critical descriptions for linearly dispersing spin-3/2 fermions are also applicable for spin-$s/2$ fermions [35]. This would imply that critical relativistic spin-1/2 fermions (describing a marginal or non-Fermi liquid) stand as an extremely sparse example of an emergent superuniversal description of the entire family of interacting quasirelativistic spin-$s/2$ fermions in two and three dimensions. This conjectured property of relativistic fermions with a half-odd-integer spin could be demonstrated numerically [20] and, possibly, in experiments. Finally, we note that the present discussion might have relevance for the observation that all fermions in the standard model are described in terms of the spin-1/2 representation, and this feature could be envisioned as an example of an emergent phenomenon, analogous to the restoration of Lorentz symmetry [36].

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[34] The rate at which the birefringent parameter β flows to zero is proportional to the bare strength of the coupling constant, such as the fine-structure constant (α) [see Eq. (6)] or the Yukawa coupling (g) [see Eq. (10)]. Whether the system flows toward the marginal or non-Fermi liquid fixed point is solely determined by the relative strength of α and g.
[35] When the long-range tail of the Coulomb interaction dominates over its short-ranged pieces, a spin-$s/2$ system flows toward a marginal-Fermi liquid fixed point of spin-1/2 fermions. This outcome does not rely on the existence of the anticommuting or mass matrix $\Gamma_{33\ldots3}$.
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