Superconductivity at an antiferromagnetic quantum critical point: Role of energy fluctuations

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Motivated by recent experiments reporting superconductivity only at very low temperature in a class of heavy fermion compounds, we study the impact of energy fluctuations with small momentum transfer on the pairing instability near an antiferromagnetic quantum critical point. While these fluctuations, formed by composite spin fluctuations, were proposed to explain the thermodynamic and transport properties near the quantum critical point of compounds such as $YbRh_2Si_2$ and $CeCu_{6-x}Au_x$ at $x \approx 0.1$, here they are found to strongly suppress T_c of the d-wave pairing of the hot quasiparticles promoted by the spin fluctuations. Interestingly, if energy fluctuations are strong enough, they can induce triplet pairing involving the quasiparticles of the cold regions of the Fermi surface. Overall, the opposing effects of energy and spin fluctuations lead to a suppression of T_c.

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I. INTRODUCTION

One of the interesting issues associated with a magnetic quantum critical point (QCP) is the possibility of superconductivity induced by the coupling between the associated quantum critical fluctuations and the electron quasiparticles [1–8]. There are a number of heavy-fermion compounds [9] that exhibit antiferromagnetic quantum criticality and superconductivity nearby in their phase diagram. Superconductivity in the cuprate [10] and iron-based [11,12] compounds is often argued to be a consequence of the presence of strong magnetic fluctuations. However, there are some prominent cases of heavy-fermion antiferromagnetic quantum criticality in which nearby superconductivity is either absent ($CeCu_{1-x}Au_x$) or has a tiny transition temperature T_c, if at all ($YbRh_2Si_2$) [13]. Elucidating why superconductivity is absent (or so fragile) in these cases, despite the presumed presence of strong magnetic fluctuations, is an important issue in the field of unconventional superconductivity.

Here, we address this issue in the framework of the recently-developed theory of critical quasiparticles whose properties are generated by their interaction with critical antiferromagnetic fluctuations [14–16]. One of the outcomes of this model is the importance of low-energy, small-momentum composite spin fluctuations, dubbed energy fluctuations [17,18]. Previously, it was shown that these energy fluctuations can explain unusual thermodynamic and transport properties observed in certain heavy fermion compounds near their magnetic QCP. In this paper, we apply an Eliashberg-like approach to investigate the interplay between spin and energy critical fluctuations to the pairing problem in a three-dimensional system.

We find that the contribution of each fluctuation channel depends strongly on the quasiparticle position on the Fermi surface (FS). It is well known that antiferromagnetic (AFM) spin fluctuations with wave-vector $\mathbf{Q}$ pair quasiparticles in the "hot line" regions of the FS, i.e., the regions for which the quasiparticle energies $\epsilon_{\mathbf{k}}$ and $\epsilon_{\mathbf{k+Q}}$ are equal [19–22]. The quasiparticles in the remaining "cold" parts of the FS are little affected. Thus, single spin fluctuation exchange can be attractive for hot quasiparticles and results in a nonzero T_c for d-wave singlet superconductivity. However, we find that the exchange of energy fluctuations is in general repulsive in that channel and may substantially reduce T_c, even to zero. On the other hand, exchange of energy fluctuations between cold quasiparticles may induce spin-triplet p-wave superconductivity, if only at a substantially lower temperature.

The paper is organized as follows: Section II reviews the strong-coupling theory of critical quasiparticles and the emergence of energy fluctuations. Section III establishes the Eliashberg-like equations to study pairing mediated by both spin and density fluctuations. These equations are then solved in Sec. IV in both singlet and triplet channels. Section V is devoted to the conclusions.

II. CRITICAL QUASIPARTICLES: NORMAL STATE PROPERTIES

In this section, we briefly outline the main results of the theoretical approach introduced in Refs. [14,15]. The usual approach for heavy-fermion metals that exhibit an antiferromagnetic quantum critical point involves consideration of the interaction of fermionic quasiparticles with the bosonic critical spin fluctuations. This may cause the fermionic degrees of freedom to also have critical behavior that acts back on the boson spectrum. This was first analyzed self-consistently in the theory of critical quasiparticles [14,15],

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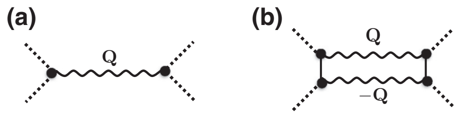

FIG. 1. Critical fluctuations: (a) Single spin fluctuation χ peaked at the AFM ordering vector $\mathbf{Q}$. (b) Structure of the energy fluctuation χ_E. A second contribution has the two spin fluctuation lines crossed. The dashed lines represent the particle and hole excitations at the Fermi surface to which the fluctuations couple. The full lines are excitations far from the Fermi surface, and the black dots represent the vertex function Λ_Q.

which was found to have two qualitatively different solutions, one in the weak-coupling and the other in the strong-coupling regime. The strong-coupling regime gives the power laws that govern transport and thermodynamic properties in the neighborhood of the QCP; it successfully accounts for experimental results in both $YbRh_2Si_2$ [16] and $CeCu_{1-x}Au_x$ [17]. In particular, it was found that the quasiparticle weight factor $Z(\omega, T) \propto [\max(\omega, T)]^{\eta} \to 0$ has a dimension-dependent fractional power of $\max(\omega, T)$. The exponents η on the cold and hot parts of the Fermi surface in the case of three-dimensional spin fluctuations were found to be $\eta_c = 1/4$ and $\eta_h = 1/2$, respectively. This leads in turn to singular critical behavior of various interaction vertex functions that are related to Z^{-1} by Ward identities [18].

The typical antiferromagnetic ordered phase is usually characterized by an ordering wave vector $\mathbf{Q}$. As discussed above, the associated critical fluctuations then connect the special hot-spot regions of the FS, which follow the condition $\epsilon_{\mathbf{k}} = \epsilon_{\mathbf{k}+\mathbf{Q}}$, where $\epsilon_{\mathbf{k}}$ is the single-electron dispersion. In three-dimensional FS, this gives rise to hot lines. As a consequence, the quasiparticle self energy generated by the exchange of such fluctuations is highly anisotropic and critical mainly at the hot spots. However, the exchange of *two* spin fluctuations with total momentum near zero [23], which may be viewed [15] as a spin exchange-energy fluctuation, gives a critical contribution over the whole FS (see Fig. 1). The critical enhancements of the interaction vertices mentioned above make such energy fluctuations important near the QCP, both for their effect on the quasiparticle self energy and for their role in superconductive pairing.

The spectrum of critical spin fluctuations is determined by the dynamical spin susceptibility

$$\chi(\mathbf{q}, \nu) = \frac{N_0}{r + (\mathbf{q} - \mathbf{Q})^2 \xi_0^2 - i\Lambda_Q^2(\nu/v_F Q)}, \tag{1}$$

where N_0 is the bare density of states at the Fermi level, r is the control parameter tuning the system through the QCP, $\xi_0 \approx k_F^{-1}$ is a microscopic correlation length, $\mathbf{Q}$ is the AFM ordering vector, $v_F = k_F/m$ is the bare Fermi velocity, and $\Lambda_Q = \Lambda(\mathbf{k}, \omega = 0; \mathbf{q}, \nu)$ is the vertex function for the antiferromagnetic spin fluctuation-particle-hole interaction, i.e., the vertex at frequency transfer ν and nonzero momentum transfer $\mathbf{q} \approx \mathbf{Q}$. Its presence in the Landau damping term of Eq. (1) reflects the feedback into the critical bosonic spin fluctuations by the critical behavior of the quasiparticles. It may be shown that when $Z^{-1}(\omega)$ diverges, then the vertex $\Lambda_Q \sim Z^{-1}$ will diverge as well. For three-dimensional spin fluctuations, $\Lambda_Q(\nu) \propto \nu^{-\eta_c}$ [18]. We note here that the static susceptibility $\chi(r, \mathbf{q}, \nu)$ diverges at $r = 0$, $\mathbf{q} = \mathbf{Q}$, and $\nu = 0$. However, at nonzero temperature, r does not diverge, i.e., the correlation length is finite, following $r \sim T^{1-2\eta}$.

We define the energy fluctuation propagator $\chi_E(\mathbf{q}, \nu)$ as the composite of two spin fluctuations with total momentum $\mathbf{q}$ near zero. The relevant diagram is shown in Fig. 1(b). Schematically, $\chi_E(\mathbf{q}, \nu) \sim \sum_{q_1,\nu_1} G \cdot G \cdot \chi(q_1, \nu_1) \cdot \chi(q_1 - q, \nu_1 - \nu)$, where one χ is peaked near $\mathbf{Q}$, the other near $-\mathbf{Q}$. The two fermion propagators G, represented by the vertical lines in the figure are both far from the FS, when the fluctuation couples to particle and hole excitations (represented by dashed lines) near the FS. The calculation, including both parallel and crossed contributions to Fig. 1(b) [15] yields

$$\mathrm{Im}\,\chi_E(\mathbf{q}, i\nu_n) \approx N_0^3 \Lambda_Q^{2d-3} \frac{|\nu_n/\gamma|^{d-1/2}}{\left[r + q^2\xi_0^2 + |\nu_n|\Lambda_Q^2/\gamma\right]^{(d+1)/2}}, \tag{2}$$

where γ is an energy scale of order the Fermi energy (e.g., $v_F Q$) and d is the dimensionality of the spin fluctuations [24]. In $d = 3$ dimensions, and on the imaginary frequency axis, the dependence of $\chi_E(\mathbf{q}, i\nu_n)$ on $\mathbf{q}, i\nu_n$ is similar to that of $\chi(\mathbf{q}, i\nu_n)$, except that χ_E diverges at $\mathbf{q} = 0$. That is,

$$\chi_E(\mathbf{q}, i\nu_n) \approx N_0^2 \Lambda_Q |\nu_n/\gamma|^{3/2} \chi(\mathbf{q} + \mathbf{Q}, i\nu_n). \tag{3}$$

The role of both χ and χ_E on the normal-state properties of the heavy fermion compounds has been investigated in Refs. [15,17]. Our goal here is to assess their interplay for the pairing instability that arises near the antiferromagnetic QCP.

III. ELIASHBERG EQUATIONS: SUPERCONDUCTING STATE PROPERTIES

To analyze the contributions of the critical fluctuations to pairing, we consider the Eliashberg-like gap equation:

$$\Phi_{\alpha\beta}(\mathbf{k}, i\omega_n) = -T \sum_{\omega_m \mathbf{p}, \gamma\delta} \frac{V_{\alpha\beta,\gamma\delta}(\mathbf{k} - \mathbf{p}, i\omega_{nm})\Phi_{\gamma\delta}(\mathbf{p}, i\omega_m)}{\omega_m^2 Z_m^{-2} + \epsilon_{\mathbf{p}}^2 + |\Phi(\mathbf{p}, i\omega_m)|^2}, \tag{4}$$

where $Z_m^{-1} = 1 - \Sigma(i\omega_m)/i\omega_m$ is the quasiparticle weight factor determined by the "second" Eliashberg equation. In this work, we will not solve the second Eliashberg equation, and instead will use the previously published results for the frequency dependence of Z in the strong-coupling regime of the model discussed above [14,15]. Here, ω_n, ω_m and $\omega_{nm} = \omega_n - \omega_m$ are fermionic and bosonic Matsubara frequencies, $\alpha, \beta, \gamma, \delta$ are spin indices and the summation over momentum $\mathbf{p}$ extends over the first Brillouin zone. As we shall only discuss the superconducting T_c, we may drop $|\Phi|^2$ in the denominator ("linearized gap equation") and eventually take Z to be the normal state quasiparticle weight. As mentioned above, Z has been calculated in Ref. [15] as $Z = (\omega/E_F)^{\eta}$, where $\eta_c = 1/4$ on the cold part of the Fermi surface and $\eta_h = 1/2$ at the hot spots.

The pairing interaction $V(\mathbf{k} - \mathbf{q}, i\omega_n - i\omega_m)$ has two contributions: one from the exchange of a single spin fluctuation, Eq. (1), the other from exchange of an energy fluctuation, Eq. (3). Both interactions are of the spin exchange type,

$$V_{\alpha\beta,\gamma\delta} = V\boldsymbol{\tau}_{\alpha\gamma}\cdot\boldsymbol{\tau}_{\beta\delta} = V_s\left(i\tau^y_{\alpha\beta}\right)\left(i\tau^y_{\gamma\delta}\right) + V_t(i\tau^y\boldsymbol{\tau})_{\alpha\beta}\cdot(i\tau^y\boldsymbol{\tau})_{\gamma\delta}, \quad (5)$$

where $\boldsymbol{\tau} = (\tau^x, \tau^y, \tau^z)$ is the vector of Pauli matrices. The last equation displays the spin dependence in the particle-particle channel. The singlet and the triplet parts are given by $V_s = 3V$ and $V_t = -V$, where

$$V(\mathbf{q}, i\nu_n) = \alpha^2\chi(\mathbf{q}, i\nu_n) + 4h(\nu_n)\alpha_E^2\chi_E(\mathbf{q}, i\nu_n). \quad (6)$$

Here, we shall approximate the coupling constants α as $\alpha \approx \Lambda_Q/N_0$ and $\alpha_E \approx \Lambda_v(\Lambda_Q/N_0)^2$. The vertex function Λ_Q at each end of a spin fluctuation was introduced below Eq. (1) and $\Lambda_v \approx Z^{-1}$ is the vertex at each end of an energy fluctuation. We have introduced the function $h(\nu_n) = [\exp(5(|\nu_n|/\nu_c - 1)) + 1]^{-1}$ which gives a soft cutoff at $\nu_c \ll \epsilon_F$ for the energy fluctuations. As for the spin fluctuations, we include the hard cutoff $\Lambda_{\text{cut}} = \epsilon_f$.

As argued in Ref. [17], for a quantum critical system to enter the strong coupling regime, as we have assumed, it is necessary that some additional quantum fluctuations, such as ferromagnetic fluctuations, should increase Z^{-1} sufficiently and actually dominate the AFM spin energy contributions when $\nu_n > \nu_c$. In the case of $YbRh_2Si_2$, the crossover from the low temperature regime, characterized by power-law behavior (e.g., specific heat coefficient $C/T \propto T^{-\eta_c}$ to the high-T behavior $C/T \propto \ln(T_0/T)$) occurs at $T \approx 0.3$ K. If we take the characteristic Fermi temperature at 10 K, we deduce a frequency cutoff $\nu_c \approx 0.03\epsilon_F$.

Although the singlet interaction is repulsive ($V_s > 0$), as is well known, the exchange of a single AF spin fluctuation that is peaked at $\mathbf{Q}$ connects quasiparticles at hot spots $\mathbf{k}_h$ and $\mathbf{k}_h + \mathbf{Q}$, which are usually far apart on the FS. This mechanism often leads to unconventional pairing of quasiparticles at the hot regions of the FS, characterized by a gap function Φ whose sign changes between these two hot spots (as would be the case for a suitable d-wave gap symmetry) [25]. Since cold quasiparticles are boosted off the FS by scattering from a single spin fluctuation, the cold regions do not contribute substantially to pairing via single spin fluctuation exchange in our scenario (see also Ref. [26]). It will be seen that exchange of energy fluctuations (peaked at $\mathbf{q} \sim 0$) gives a *repulsive* contribution to the pairing kernel, as it connects $\mathbf{k}_h + \mathbf{q} \approx \mathbf{k}_h$ for which the gap function has the *same* sign. Therefore, we investigate below the suppression, by energy fluctuations, of d-wave singlet superconductivity from the hot regions.

As well as being repulsive in the singlet channel, the exchange of energy fluctuations in the triplet channel will be attractive provided it couples close regions of the FS (as it does, since $q \approx 0$) for which the gap function does *not* change sign

$$V_t = -4h(\nu_n)\alpha_E^2\chi_E(\mathbf{q}, i\nu_n). \quad (7)$$

This pairing interaction is equally strong over the whole FS and so could lead to triplet pairing of cold quasiparticles. The orbital symmetry of the resulting gap function will likely be the most symmetric form compatible with the requirement of odd parity imposed by the Pauli principle, e.g., p-wave pairing in the present case.

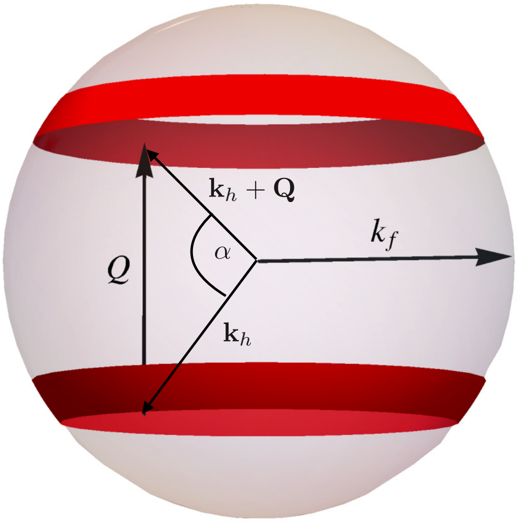

FIG. 2. Schematic plot of the hot lines (red) on the Fermi surface. The hot lines satisfy the condition $\epsilon_{\mathbf{k}_h} = \epsilon_{\mathbf{k}_h+\mathbf{Q}} = 0$, where $\mathbf{Q}$ is the antiferromagnetic wave vector. Note that the hot lines have a finite width.

IV. CALCULATION OF T_c

For the actual solution of the linearized gap equation, we take a simple isotropic model of a three-dimensional metal with dispersion $\epsilon_{\mathbf{k}} \approx v_F(k - k_F)$ and three-dimensional antiferromagnetic fluctuations as is appropriate for $YbRh_2Si_2$. The spherical FS has lines of hot spots $\mathbf{k}_h$, where $\epsilon_{\mathbf{k}_h} = \epsilon_{\mathbf{k}_h+\mathbf{Q}} = 0$. Figure 2 shows the two hot lines on the FS (in red) that are connected by the AFM vector $\mathbf{Q}$ taken here to be parallel to the z axis. The hot lines are located at polar angle $\theta_0 = \cos^{-1}(Q/2k_F)$ and at $\pi - \theta_0$. The width of the hot lines [15] depends on the temperature as $\delta\theta \approx \Lambda_Q\sqrt{T/\epsilon_F}\sin\alpha$, where $\alpha = \pi - 2\theta_0$ is the angle between the quasiparticle velocities $\mathbf{v}_{\mathbf{k}_h}$ and $\mathbf{v}_{\mathbf{k}_h+\mathbf{Q}}$, see Fig. 2.

A. Hot quasiparticles

As explained earlier, we will restrict the analysis of singlet pairing to the neighborhood of the hot lines. The linearized gap equation has the form

$$\Phi(\mathbf{k}, i\omega_n) = -3T\sum_{\omega_m;\mathbf{p};\gamma,\delta}\frac{V(\mathbf{k}-\mathbf{p}, i\omega_{nm})\Phi(\mathbf{p}, i\omega_m)}{\omega_m^2 Z_m^{-2} + \epsilon_{\mathbf{p}}^2}, \quad (8)$$

where

$$V(\mathbf{q}, i\nu_n) = \alpha^2\chi(\mathbf{q}, i\nu_n) + 4h(\nu_n)\alpha_E^2 N_0^2\Lambda_Q|\nu_n|^{3/2}\chi(\mathbf{q}+\mathbf{Q}, i\nu_n)\,, \quad (9)$$

where the second term comes from χ_E in Eq. (3). Neglecting the dependence of the gap function on $|\mathbf{p}|$, its dependence is only on the polar angle θ and the azimuthal angle ϕ since $\mathbf{p}$ is on the FS, i.e., $|\mathbf{p}| = k_F$. On one hand, the Pauli principle requires the gap function on the hot lines ($\theta = \theta_0$ and $\theta = \pi - \theta_0$) to obey $\Phi(\theta_0, \phi, i\omega_m) = \Phi(\pi - \theta_0, \pi + \phi, i\omega_m)$. On the other hand, as explained above, the gap must change sign between the two hot lines in order to solve the gap equation, $\Phi(\mathbf{p}, i\omega_m) \approx -\Phi(\mathbf{p} + \mathbf{Q}, i\omega_m)$. Combining these two conditions yields $\Phi(\theta_0, \phi, i\omega_m) = -\Phi(\theta_0, \pi + \phi, i\omega_m)$. We therefore look for a solution of the form $\Phi(\theta, \phi, i\omega_m) = \Delta_m^s \cos\theta \cos\phi$ defined along the hot lines.

In the first term of the effective interaction in Eq. (9), we may shift $\mathbf{p} \to \mathbf{p} - \mathbf{Q}$, which leaves the factor $(\omega_m^2 Z_m^{-2} + \epsilon_{\mathbf{p}}^2)^{-1}$ invariant, since $\epsilon_{\mathbf{p}-\mathbf{Q}} = \epsilon_{\mathbf{p}}$ on the hot lines, while the sign of Φ changes. We choose the proper sign of $\mathbf{Q}$, depending on whether $\mathbf{p}$ is on the upper or lower hot line (see Fig. 2). Therefore, the interaction function simplifies to $V(\mathbf{q}, i\nu_n) \to [-\alpha^2 + 4h_{nm}\alpha_E^2 N_0^2 \Lambda_Q |\nu/\gamma|^{3/2}]\chi(\mathbf{q} + \mathbf{Q}, i\nu_n)$.

The linearized gap equation then takes the form

$$\Delta_n^s \cos\theta_k \cos\phi_k = 3T \sum_{\omega_m;\mathbf{p}} \frac{N_0}{r + (\mathbf{k} - \mathbf{p})^2/k_F^2 + \Lambda_{Q,nm}^2 |\omega_{nm}|} \times \left[\alpha_{nm}^2 - 4h_{nm}\alpha_{E;nm}^2 N_0^2 \Lambda_{Q,nm} |\omega_{nm}|^{3/2}\right] \times \frac{\Delta_m^s \cos\theta_p \cos\phi_p}{\omega_m^2 Z_m^{-2} + \epsilon_{\mathbf{p}}^2}, \tag{10}$$

Here, the momentum integration is restricted to the hot lines. For not too large $|\omega_m| \ll \epsilon_F$ the factor $[\omega_m^2 Z_m^{-2} + \epsilon_{\mathbf{p}}^2]^{-1}$ is sharply peaked at $p = k_F$, so that one may write $(\mathbf{k} - \mathbf{p})^2 = 2k_F^2(1 - \cos\theta_{kp})$, where θ_{kp} is the angle enclosed by $(\mathbf{k}, \mathbf{p})$. If we take $\mathbf{k} = k_F(\sin\theta_0, 0, -\cos\theta_0)$ on the lower hot line, we have $(\mathbf{k} - \mathbf{p})^2 = 2k_F^2[1 + \cos\theta_0 \cos\theta_p - \sin\theta_0 \sin\theta_p \cos\phi_p]$.

The integration over the angles ϕ_p and θ_p as well as the integration over $\epsilon_{\mathbf{p}}$ can all be done analytically. This results in a matrix equation in frequency space:

$$\Delta_n^s = \frac{3\pi T_c}{2} \sum_m W_{nm}^s \frac{\Delta_m^s}{|\omega_m|/Z_m}, \tag{11}$$

where

$$W_{nm}^s = \left[\Lambda_{c,nm}^2 - 4h_{nm}\Lambda_{c,nm}^5 \Lambda_{h,nm}^2 |\Omega_{nm}|^{3/2}\right] I_{nm}^s \tag{12}$$

with

$$I_{nm}^s = \frac{1 + B_{nm}^s}{\sqrt{1 + B_{nm}^s/2}} \sinh^{-1} \frac{\delta\theta}{\sqrt{2\sin^2\theta_0 B_{nm}^s}} - \frac{\delta\theta}{\sin\theta_0},$$
$$B_{nm}^s = \Lambda_{c,nm}^2 |\Omega_{nm}|/2\sin^2\theta_0 \tag{13}$$

and $h_{nm} = h(\omega_n - \omega_m)$ is the soft cutoff function introduced earlier. It is convenient to define $f_m^s = \Delta_m^s Z_m/|\omega_m|$ and to re-express the gap equation as the matrix eigenvalue equation [27]

$$\sum_m K_{n,m}^s f_m^s = 0, \tag{14}$$

where the kernel is given by

$$K_{n\neq m}^s = \tfrac{3}{2} W_{nm}^s,$$
$$K_{n,n}^s = -(2n+1)Z_n^{-1} + \tfrac{1}{2}\left(K_{n,n-1}^s + K_{n-1,n}^s\right). \tag{15}$$

Here, the subscript nm stands for the frequency difference $\omega_n - \omega_m$. We have regularized the weak singularity of $K_{n,m}^s$ in the limit $n \to m$, which is cutoff by temperature as noted in the text below Eq. (1), by setting $K_{n,n}^s \approx \frac{1}{2}(K_{n,n-1}^s + K_{n-1,n}^s)$. The subscripts c, h label cold or hot quasiparticle quantities. Following Ref. [15], we set $\Lambda_Q = \Lambda_{c,nm} = Z_{c,nm}^{-1} = 1 + \Lambda_c^{(0)}|\omega_n - \omega_m|^{-1/4}$ and $\Lambda_v = \Lambda_{h,nm} = Z_{h,nm}^{-1} = 1 + \Lambda_h^{(0)}|\omega_n - \omega_m|^{-1/2}$ on the hot lines, but $\Lambda_v = \Lambda_{c,nm}$ on the cold parts of the Fermi surface. The parameters $\Lambda_c^{(0)}, \Lambda_h^{(0)}$ will be considered as tuning parameters controlling the strength of the fluctuations.

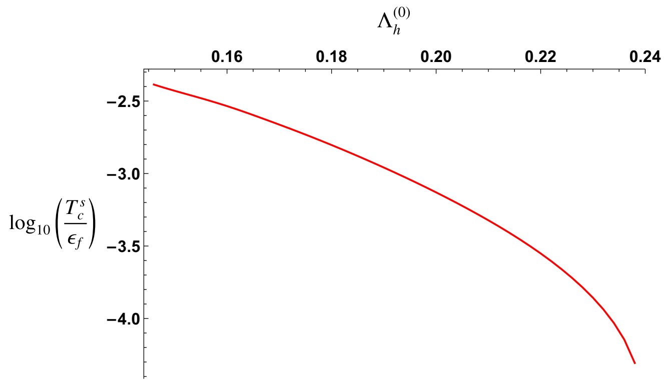

FIG. 3. Suppression of T_c by energy fluctuations. The pairing channel considered here is the singlet channel, promoted by the exchange of spin fluctuations between hot quasiparticles. $\Lambda_h^{(0)}$ denotes the strength of the energy-fluctuation vertex for the hot quasiparticles. Here, ϵ_F is the Fermi energy.

To assess the impact of energy fluctuations on the T_c for singlet pairing of the hot quasiparticles, we tune the hot vertex prefactor $\Lambda_h^{(0)}$, a measure of the strength of hot pairing, from $\Lambda_h^{(0)} \approx 0.15$ up to 0.24. These particular values are chosen because for $\Lambda_h^{(0)} < 0.15$, $2\pi T_c$ is above the energy cutoff of the energy fluctuation, whereas for $\Lambda_h^{(0)} > 0.24$, T_c is below our numerical precision. Note that, because $\Lambda_h^{(0)}$ only affects Λ_v, and because the contribution to the pairing interaction arising from the energy fluctuations has an overall Λ_v prefactor [see the α_E term in Eq. (6)], by changing $\Lambda_h^{(0)}$ we are effectively changing the relative strength of the energy fluctuations over the spin fluctuations. The strength of the cold vertex is kept fixed as $\Lambda_c^{(0)} = 0.5$. In addition, the AFM vector $\mathbf{Q} = \sqrt{2}k_f$ and thus $\theta_0 = \pi/4$.

The resulting T_c is plotted in Fig. 3. When the energy fluctuations contribution is weaker ($\Lambda_h^{(0)} = 0.15$), a nonzero T_c of order $0.004\epsilon_F$ is found at the QCP ($r = 0$). However, when the energy fluctuations contribution becomes stronger, T_c suffers a substantial suppression. This is in agreement with experiments in $YbRh_2Si_2$, where superconductivity appears to be absent in the expected temperature range of several hundreds of mK. Another compound for which energy fluctuations are thought to exist is $CeCu_{6-x}Au_x$ at $x \approx 0.1$, where again superconductivity has not been observed. In the latter, two-dimensional antiferromagnetic spin fluctuations are thought to dominate and a model calculation analogous to the one presented above applies. It is also interesting to study T_c without the contribution of the energy fluctuations, i.e., $Z_h = 1$, which removes the T_c suppression arising from the energy fluctuations from Eq. (10). In this case, we found that $T_c/\epsilon_f \approx 0.024$, or $T_c \approx 0.24$ K, suggesting that a strong suppression of T_c by energy fluctuations is present in these compounds.

B. Cold quasiparticles

As discussed in Eq. (7), a triplet pairing interaction is also generated by the exchange of energy fluctuations. We assume p-wave symmetry as discussed above and consider the gap function of the form $\Phi(\mathbf{k}, i\omega_n) = \Delta_n^t \cos\theta$, where θ is the angle between $\mathbf{k}$ and the z axis. The linearized gap equation becomes

$$\Delta_n^t \cos\theta_k = T \sum_{\omega_m;\mathbf{p}} \frac{4N_0^3 h_{nm}\alpha_{E,nm}^2 \Lambda_{Q,nm}|\omega_{nm}|^{3/2}}{r + (\mathbf{k}-\mathbf{p})^2 + \Lambda_{Q,nm}^2|\omega_{nm}|} \quad (16)$$

$$\times \frac{\Delta_m^t \cos\theta_p}{\omega_m^2 Z_m^{-2} + \epsilon_{\mathbf{p}}^2}. \quad (17)$$

In contrast to the case of hot quasiparticles, the vertex function for the cold quasiparticles is $\lambda_v = \Lambda_c$, resulting in the coupling constant $\alpha_E \approx \Lambda_c(\Lambda_Q/N_0)^2$. Performing the momentum integral in a similar way as in the singlet pairing case and again defining $f_m^t = \Delta_m^t Z_m/|\omega_m|$, the following eigenvalue problem in Matsubara frequency space is found:

$$\sum_{\omega_m} K_{n,m}^t f_m^t = 0. \quad (18)$$

The kernel is given by

$$K_{n\neq m}^t = \tfrac{1}{2} W_{nm}^t,$$
$$K_{n,n}^t = -(2n+1)Z_n^{-1} + \tfrac{1}{2}\left(K_{n,n-1}^t + K_{n-1,n}^t\right), \quad (19)$$

where

$$W_{nm}^t = 4h_{nm}\Lambda_{c,nm}^7|\Omega_{nm}|^{3/2} I_{nm}^t \quad (20)$$

and

$$I_{nm}^t = \left(2 + B_{nm}^t\right)\ln\left(1 + 4/B_{nm}^t\right) - 4,$$
$$B_{nm}^t = \Lambda_{c,nm}^2|\Omega_{nm}|. \quad (21)$$

Again, $h_{nm} = h(\omega_n - \omega_m)$ is the soft cutoff function introduced above.

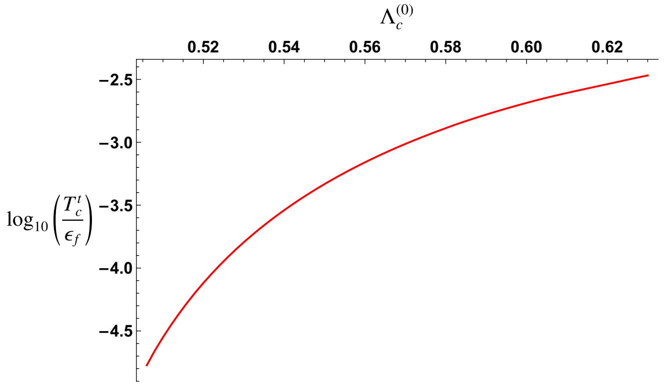

FIG. 4. Superconducting transition temperature T_c^t in the triplet pairing channel. This pairing is mediated by the exchange of energy fluctuations by cold quasiparticles. $\Lambda_c^{(0)}$ denotes the strength of the energy fluctuation vertex for cold quasiparticles.

The T_c values for triplet pairing obtained by numerical solution of Eq. (18) are shown in Fig. 4 as a function of the bare vertex strength $\Lambda_c^{(0)}$. To keep T_c in the numerically accessible range, i.e., above our numerical resolution and below the cutoff, we constrain $\Lambda_c^{(0)}$ to the range plotted in the figure. A strong dependence on $\Lambda_c^{(0)}$ is found. In particular, for the value $\Lambda_c^{(0)} = 0.5$ that we chose for the singlet pairing solution, we find $T_c/\epsilon_F \approx 1.5 \times 10^{-5}$, corresponding to $T_c \approx 0.15$ mK, as compared to the singlet pairing $T_c \approx 0.24$ K found in the absence of energy fluctuations. It remains to be seen whether the superconducting phase observed [13] in $YbRh_2Si_2$ at mili-Kelvin temperatures is of spin-triplet symmetry, which our calculations suggest to be a possibility.

V. CONCLUSIONS

Motivated by recent experimental evidence [13] for superconductivity at extremely low temperature in $YbRh_2Si_2$, we have used the recently-developed theory of critical quasiparticles [14,15] to discuss the superconductivity generated by pairing mediated by critical fluctuations in the neighborhood of an antiferromagnetic quantum critical point, which is often present in the phase diagram of heavy-fermion compounds. In these materials, critical antiferromagnetic spin fluctuations are dominant and are responsible for many of the observed properties near the critical region. Since these fluctuations have a nonzero wave vector $\mathbf{Q}$, usually of order k_F, they divide the Fermi surface into hot regions, which are connected by $\mathbf{Q}$, and cold regions, which are not. This usually leads to unconventional pairing (e.g., d-wave) of hot quasiparticles as is seen in cuprates and some heavy-fermion superconductors.

However, as emphasized in Refs. [15,17,23], composite critical spin fluctuations induce energy fluctuations at small momentum, leading to a diverging quasiparticle effective mass over the *whole* Fermi surface. This contribution is essential to achieve the excellent agreement between the critical quasiparticle theory with the experimental results for thermodynamic and transport quantities on $CeCu_{1-x}Au_x$ and $YbRh_2Si_2$. In this paper, we studied the impact of these energy fluctuations on the pairing channel by employing an Eliashberg-like approach. Our main results are that, while the exchange of energy fluctuations suppresses the d-wave T_c of hot quasiparticles, they can at the same time mediate spin-triplet (e.g., p-wave) superconductivity of cold quasiparticles, a possibility that can be probed experimentally, for example using NMR.

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