Developing Fractional Quantum Hall States at Even-Denominator Fillings 1/6 and 1/8

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In the extreme quantum limit, when the Landau level filling factor $\nu < 1$, the dominant electron-electron interaction in low-disorder two-dimensional electron systems leads to exotic many-body phases. The ground states at even-denominator $\nu = 1/2$ and $1/4$ are typically Fermi seas of composite fermions carrying two and four flux quanta, surrounded by the Jain fractional quantum Hall states (FQHSs) at odd-denominator fillings $\nu = p/(2p \pm 1)$ and $\nu = p/(4p \pm 1)$, where p is an integer. For $\nu < 1/5$, an insulating behavior, which is generally believed to signal the formation of a pinned Wigner crystal, is seen. Our experiments on ultra-high-quality, dilute, GaAs two-dimensional electron systems reveal developing FQHSs at $\nu = p/(6p \pm 1)$ and $\nu = p/(8p \pm 1)$, manifested by magnetoresistance minima superimposed on the insulating background. In stark contrast to $\nu = 1/2$ and $1/4$, however, we observe a pronounced, sharp minimum in magnetoresistance at $\nu = 1/6$ and a somewhat weaker minimum at $\nu = 1/8$, suggesting developing FQHSs, likely stabilized by the pairing of composite fermions that carry six and eight flux quanta. Our results signal the unexpected entry, in ultra-high-quality samples, of FQHSs at *even-denominator* fillings 1/6 and 1/8, which are likely to harbor non-Abelian anyon excitations.

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When a two-dimensional electron system (2DES) at low temperatures is subjected to a large, perpendicular magnetic field (B), the electrons' kinetic energy is quenched as they occupy quantized, dispersionless Landau levels (LLs). The dominant electron-electron Coulomb energy leads to a variety of exotic, strongly correlated, many-body ground states, depending on the LL filling factor $\nu = nh/eB$, where n is the 2DES density. When ν is a rational fraction, fractional quantum Hall states (FQHSs), incompressible liquid states that host quasiparticles with fractional charge and anyonic statistics, manifest as the ground states [1–5]. Of particular interest are FQHSs observed at *even-denominator* fillings of excited-state ($N = 1$) LLs, e.g., at $\nu = 5/2$ [6], because they are likely to harbor non-Abelian anyon excitations [7,8], which can be useful for topological quantum computation [9]. However, the vast majority of FQHSs are observed in the extreme quantum limit ($\nu < 1$) at *odd-denominator* fillings, and are successfully explained by Laughlin's wave function [10] and Jain's composite fermion (CF) theory [2,11].

Another longstanding, fundamental, and important question relates to the fate of a clean 2DES at extremely small ν. It is generally believed that, for sufficiently small ν, electrons should form an ordered array, known as the Wigner crystal (WC) [12,13]. Theorists predict a termination of the FQHSs and transition into a quantum WC at a critical ν ranging from 1/6 to 1/11 [10,14–17]. On the experimental front, strong evidence for pinned WC states was reported at $\nu \lesssim 1/5$ in GaAs 2DESs [18–22]. The WC exhibits an insulating behavior because of the pinning by the ubiquitous disorder in a realistic (nonideal) 2DES. In the highest quality samples, signatures of FQHSs were also reported in the very small filling regime, e.g., at $\nu = 1/7$, in the form of resistance minima superimposed on the strongly insulating background [23–25]. These observations highlight the very close competition between the WC and FQHS phases.

In this Letter, we examine the regime of $\nu \ll 1$ in ultra-high-quality GaAs 2DESs. We observe an unexpected emergence of new correlated states deep in the WC regime, namely *even-denominator* FQHSs at $\nu = 1/6$ and 1/8. These states are likely non-Abelian FQHSs stabilized by the pairing of six-flux and eight-flux CFs (^{6}CFs and ^{8}CFs). The presence of these large-flux CFs is also evinced by the observation of numerous odd-denominator FQHSs on the flanks of $\nu = 1/6$ and 1/8, following the Jain sequence of ^{6}CFs and ^{8}CFs.

We studied high-quality 2DESs confined to GaAs quantum wells (QWs) grown on GaAs (001) substrates by molecular beam epitaxy. They were grown following the optimization of the growth chamber vacuum integrity and the purity of the source materials [26]. We used 4×4 mm^2 van der Pauw geometry samples with alloyed In:Sn contacts at the four corners and side midpoints. The samples were cooled in a dilution refrigerator. We measured the longitudinal resistances (R_{xx}) using the conventional lock-in amplifier technique.

As highlighted in Fig. 1(a), on the flanks of $\nu = 1/6$, we observe a sequence of minima at $\nu = 1/5$, 2/11, 3/17, and 1/7, 2/13, 3/19, superimposed on an extremely large and

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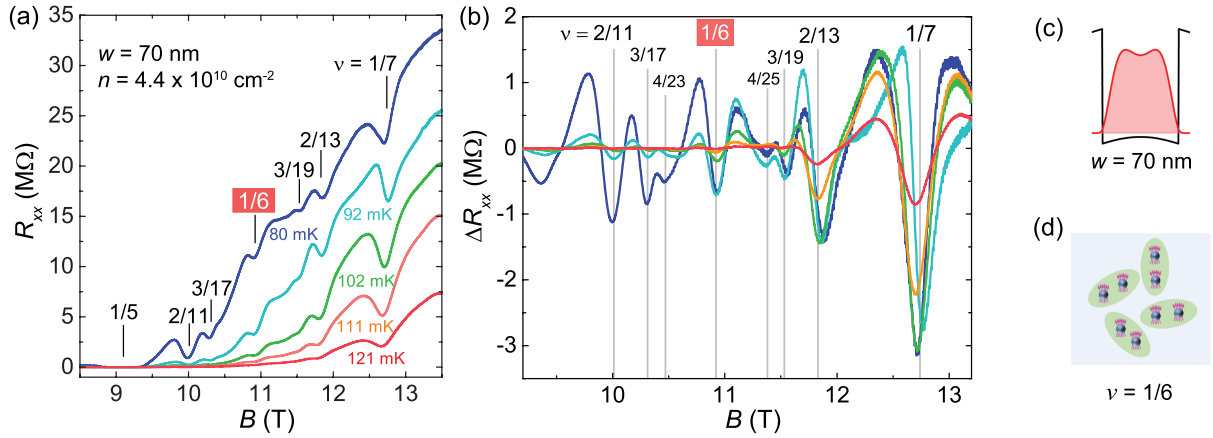

FIG. 1. (a) Longitudinal resistance (R_{xx}) vs perpendicular magnetic field (B) traces for our ultra-high-mobility 2DES in the extremely small filling regime ($1/5 > \nu > 1/7$), measured at different temperatures [27]. The 2DES is confined to a 70-nm-wide QW, and has a density of 4.4×10^{10} cm^{-2} and a record mobility of 22×10^6 cm^2/Vs at this density. The magnetic field positions of several LL fillings are marked. Our data exhibit numerous local minima in R_{xx} at odd-denominator fillings 1/5, 2/11, 3/17, 1/7, 2/13, and 3/19. These fillings correspond to the Jain-sequence states of six-flux CFs (^{6}CF). Remarkably, we also observe a local minimum in R_{xx} at the *even-denominator* filling $\nu = 1/6$, suggesting a developing FQHS. (b) ΔR_{xx} vs B traces for the same set of data, where ΔR_{xx} is the resistance after subtracting the increasing, smooth background [28]. (c) Self-consistent charge distribution (red) and potential (black) for the 2DES. (d) A possible origin of the 1/6 FQHS: each electron captures six flux quanta to turn into a ^{6}CF. Then ^{6}CFs undergo a pairing instability and condense into a FQHS.

insulating R_{xx} [27]. These are the Jain-sequence FQHSs of ^{6}CFs [$\nu = p/(6p \pm 1)$], emanating from $\nu = 1/6$, analogous to the standard Jain-sequence FQHSs of ^{2}CFs and ^{4}CFs observed on the flanks of CF Fermi seas at $\nu = 1/2$ and 1/4; see Fig. S4 in Supplemental Material (SM) [28] for data near $\nu = 1/2$ and 1/4. Our observation is consistent with recent calculations that indicate that, in the low-disorder limit, Jain-sequence FQHSs of ^{6}CFs should prevail at $\nu = 1/7$ and 2/13 [17]. The appearances of the very-high-order FQHSs near $\nu = 1/2$ and 1/4, and the rarely observed FQHSs near $\nu = 1/6$ [23–25], collectively demonstrate the exceptionally high quality of our 2DES, specially at such a low density ($n = 4.4$, in units of 10^{10} cm^{-2}, which we use throughout the Letter).

Our main finding is the pronounced, sharp minimum in R_{xx} at the *even-denominator* filling $\nu = 1/6$. As we illustrate below, the characteristics of this minimum are very similar to those of the nearby, emerging, odd-denominator FQHSs. Our data signal a developing even-denominator FQHS at $\nu = 1/6$, likely stabilized by the pairing of ^{6}CFs.

Figure 1(a) also shows high-field R_{xx} vs B traces at different temperatures. As T increases from 80 to 121 mK, R_{xx} at $\nu < 1/5$ decreases by more than an order of magnitude. Meanwhile, R_{xx} minima at $\nu = 1/6$ and $\nu = p/(6p \pm 1)$ gradually weaken and eventually turn into inflection points [28,39]. To highlight FQHS features, in Fig. 1(b), we present ΔR_{xx} vs B traces, with ΔR_{xx} representing resistance after subtracting the smooth background; see Sec. I of SM for details [28]. We observe sharp ΔR_{xx} minima at $\nu = p/(6p \pm 1)$ for $p = 1, 2, 3, 4$, and at $\nu = 1/6$. The $\nu = p/(6p \pm 1)$ minima are weaker for larger p and weaken with increasing T, consistent with standard Jain-sequence FQHSs. The $\nu = 1/6$ minimum is sharp and exhibits similar temperature dependence to those at Jain-sequence fillings, signaling a developing FQHS at $\nu = 1/6$.

We note that with decreasing temperature, instead of approaching zero, R_{xx} at $\nu = 1/6$ and $p/(6p \pm 1)$ increases. This is because an insulating behavior, which is a manifestation of a pinned WC [18–22], is dominant in the whole range of $\nu < 1/5$. Our observation signals a close competition between the FQHSs and WC states. More specifically, the energies of WC and FQHSs are so close that FQHSs only win in a very narrow range of ν [17]. Therefore, in a realistic 2DES, a small local variation of filling factor caused by a minuscule density inhomogeneity or disorder can lead to the formation of WC domains and prevent the percolation of the fractional quantum Hall (FQH) liquid [40]. Our data are reminiscent of what was historically observed at $\nu = 1/5$ in GaAs 2DESs [19,20,37,38,41,42]. Initially, in modest-quality samples, only an R_{xx} minimum that rose with decreasing temperature was seen because of the significant amount of disorder [41,42]. With improved sample quality, percolation of the FQH liquid was eventually achieved, exhibiting a vanishing R_{xx} accompanied by a quantized Hall plateau, firmly establishing that the ground state at $\nu = 1/5$ is a FQHS [19,37,38].

We measured a second sample from the same wafer [43]. Figure 2(a) shows the R_{xx} vs $1/\nu$ traces measured at $T \simeq 80$ mK with n ranging from 2.77 to 5.10, while maintaining symmetric charge distribution. We observe a

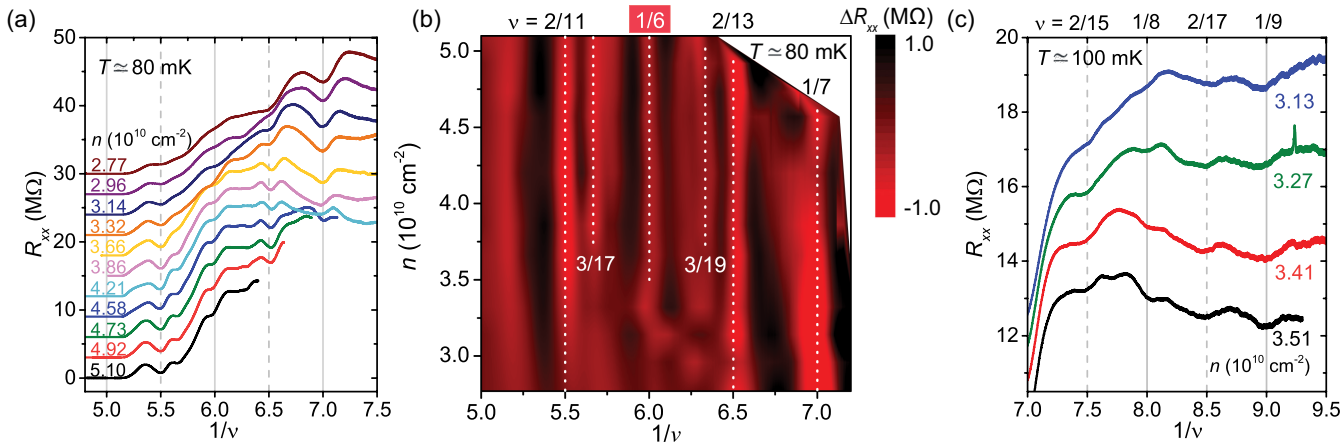

FIG. 2. Density dependence. (a) R_{xx} vs $1/\nu$ traces measured at $T \simeq 80$ mK for different densities n. Each trace is vertically shifted by 3 MΩ for clarity. n is tuned by symmetrically gating the 2DES from both the top and bottom. (b) Color-scale plot of ΔR_{xx} as a function of $1/\nu$ and n. Several fillings are marked by white dotted lines. (c) R_{xx} vs $1/\nu$ traces measured at $T \simeq 100$ mK and at very small ν. R_{xx} minima are observed at $\nu = 1/9$, $2/17$, $1/8$, and $2/15$.

clear inflection point at $\nu = 1/6$. It becomes weaker at lower densities [44]. In addition, well-defined R_{xx} minima are observed at $\nu = 2/11$ in the whole range of n, and at $\nu = 1/7$ in the lower density traces where we could reach $\nu = 1/7$ with our magnet. Figure 2(b) displays a color-scale plot of ΔR_{xx} as a function of $1/\nu$ and n. Distinct ΔR_{xx} minima are observed at $\nu = 2/11$, $3/17$, $3/19$, $1/6$, $2/13$, and $1/7$, with their $1/\nu$ positions remaining consistent across different n, indicating that the signatures of the even-denominator FQHS at $\nu = 1/6$, as well as the high-order Jain-sequence FQHSs at $\nu = p/(6p \pm 1)$, are intrinsic to our ultra-high-quality 2DES.

Figure 2(c) data show the fate of the FQHSs at yet smaller ν. Here, we observe R_{xx} minima at odd-denominator $\nu = 1/9$, $2/17$, and $2/15$. Hints of developing FQHSs were previously reported at $\nu = 1/9$ by optical and transport measurements [24,45]. Recent calculations also suggest that the ground state at $\nu = 1/9$ is likely a FQHS [17]. Our data revealing R_{xx} minima at fixed fillings over a range of densities provide strong evidence for the existence of FQHSs at $\nu = p/(8p \pm 1)$, namely at $\nu = 1/9$, $2/17$, and $2/15$ [46]. Furthermore, the traces also exhibit an R_{xx} minimum (or an inflection point) at the *even-denominator* filling $\nu = 1/8$. The qualitative resemblance of the data near $\nu = 1/8$ in Fig. 2(c) to Fig. 2(a) data suggests that the physics for ^{6}CFs for $1/7 < \nu < 1/5$ can be extended to ^{8}CFs for $1/9 < \nu < 1/7$.

The signatures of the $\nu = 1/6$ FQHS are not specific to one wafer. In Fig. 3 we present data for another ultra-high-quality GaAs 2DES from a different wafer with a higher density of 7.1 and a narrower QW width of 58.5 nm. We find a clear, sharp R_{xx} minimum at $\nu = 1/6$ at 108 mK [47], and an inflection point at a slightly higher temperature of 116 mK.

One potential competitive ground state at $\nu = 1/6$ is a metallic Fermi sea of ^{6}CFs at zero effective magnetic field. However, several observations suggest that the R_{xx} minimum we observe at $\nu = 1/6$ is not indicative of a ^{6}CF Fermi sea: (i) At $\nu = 1/2$, where a Fermi sea of ^{2}CFs is well established, typically a smooth and broad minimum in R_{xx} is observed; see, e.g., Figs. S4 and S5 in SM [28]. In contrast, the R_{xx} minimum at $\nu = 1/6$ is sharp; see Figs. 1 and 3, and also the sharp peak in d^2R_{xx}/dB^2 [Fig. S2(b)]. The sharp R_{xx} minimum observed in the WC regime at $\nu = 1/6$ indicates that the $\nu = 1/6$ state is flanked by WC states. This strongly favors the interpretation of FQHS over

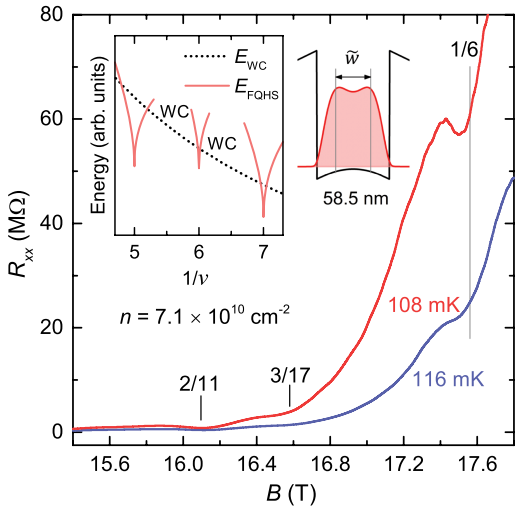

FIG. 3. Data for a different sample. R_{xx} vs B traces for a sample with a density of 7.1×10^{10} cm^{-2}. Left inset: schematic diagram showing the energies of WC and FQHSs vs $1/\nu$, indicating the possibility of a downward cusp in energy at $1/\nu = 6$, similar to cusps at $1/\nu = 5$ and 7. Right inset: self-consistent charge distribution (red) and potential (black) for the 2DES confined to a 58.5-nm-wide QW; $\tilde{w}$ denotes the electron layer thickness (see text).

TABLE I. Sample parameters.

Sample	Density (10^{10} cm^{-2})	Mobility (10^6 cm^2/Vs)	QW width (nm)	$\tilde{w}/l_B$ (at $\nu = 1/6$)	$\Delta_{\rm SAS}/E_{\rm Coul}$ (at $\nu = 1/6$)	Strength of $\nu = 1/6$ FQHS feature
A	4.4	22	70	4.3	0.13	Strong
B	7.1	25	58.5	4.6	0.15	Strong
C	4.3	16	50	2.7	0.33	Weak
D	4.5	17	40	2.2	0.52	Weak
E	4.3	10	30	1.7	0.93	Absent

^{6}CF Fermi sea: the energies of FQHSs show downward cusps as a function of ν, and can dip below the WC energy (see Fig. 3 left inset, and also see Fig. 9 in Ref. [48]) so that the WC could be reentrant on the flanks of a FQHS [17,19,33]. On the other hand, the energies of both CF Fermi sea and WC states are smooth functions of ν, and thus one would expect only a single transition between these two states. (ii) The R_{xx} minimum at $\nu = 1/6$ and its temperature dependence closely resemble those at Jain-sequence, odd-denominator fillings, such as $\nu = 1/7$ (see Fig. 1 and also Fig. S6 in SM [28,39]), challenging the metallic ^{6}CF Fermi sea interpretation. (iii) A broad R_{xx} minimum is not a universal feature for a CF Fermi sea, and is indeed generally absent at $\nu = 1/4$ when there is a competing insulating phase or rising background resistance [49]; also see SM [28]. It is also worth noting that the observation of Jain-sequence FQHSs does not necessarily indicate the presence of a CF Fermi sea. Indeed, a FQHS at $\nu = 1/2$ flanked by numerous Jain-sequence FQHSs is well established in 2DESs confined to wide GaAs QWs [50–56]. We emphasize that, while we believe our data favor the presence of a developing FQHS at $\nu = 1/6$, taking the R_{xx} minimum as evidence for a ^{6}CF Fermi sea at 1/6 is also novel as there has been no experimental or theoretical evidence for such a phase deep in the low-filling, insulating WC regime.

The emergence of even-denominator FQHSs at $\nu = 1/6$ and 1/8 is unexpected. These states have not been reported in any 2D carrier system, or predicted by existing theories. One might wonder if these new enigmatic states share a common origin with two other even-denominator FQHSs, namely those at $\nu = 1/2$ and 1/4, observed in the lowest LL in wide QWs under special circumstances [50–57]. The origin of these states has been a subject of debate [58]; however, recent experiments [54–56] and theories [59–61] indicate they are likely single-component, non-Abelian states arising from a pairing instability of CFs. To examine this possibility and investigate the role of QW width on the 1/6 FQHS, we studied several samples with similar densities $n = 4.4$, 4.3, 4.5, and 4.3, but different QW widths of 70, 50, 40, and 30 nm, respectively. Below we summarize our findings; for details, see Sec. IV in SM [28].

Table I contains the relevant parameters for our samples. One parameter is the effective electron layer thickness ($\tilde{w}$), normalized to the magnetic length $l_B = \sqrt{\hbar/eB}$; $\tilde{w}$ is typically defined as twice the standard deviation of the calculated charge distribution from its center; see Fig. 3 inset for an example. Another parameter is the symmetric-to-antisymmetric subband separation ($\Delta_{\rm SAS}$) normalized to the Coulomb energy, $E_{\rm Coul} = e^2/(4\pi\epsilon l_B)$. For 2DESs confined to QWs, in the extreme quantum limit ($\nu < 1$), although only one LL that originates from the symmetric subband is partially occupied, the proximity of the antisymmetric subband can potentially modify the electron-electron interaction in the lowest LL when $\Delta_{\rm SAS}/E_{\rm Coul} \ll 1$. As seen in Table I, we observe the strongest $\nu = 1/6$ FQHS in samples A and B where $\tilde{w}/l_B$ is large ($\gtrsim 4$) and $\Delta_{\rm SAS}/E_{\rm Coul}$ is small ($\lesssim 0.2$), suggesting that the $\nu = 1/6$ FQHS is stabilized by the large layer thickness and proximity of the antisymmetric subband.

While the 1/2 and 1/4 FQHSs are also seen at large $\tilde{w}/l_B$ and reasonably small $\Delta_{\rm SAS}/E_{\rm Coul}$, the 1/6 data pose several notable quantitative and qualitative differences. First, the parameters for the samples where we observe the strong $\nu = 1/6$ minima are different compared to those where the 1/2 and 1/4 FQHSs are seen. This is visually exhibited in "phase diagrams" for the stability of the $\nu = 1/2$ FQHS in wide GaAs QWs shown in Fig. S9 in SM [28]. Second, the 1/2 and 1/4 FQHSs are observed only in samples with *bilayer* charge distributions [50–57], but in our samples the charge distribution is single-layer, albeit thick [see Figs. 1(c) and 3 inset]. Third, the 1/2 and 1/4 FQHSs in wide QWs become weaker and disappear when the charge distribution is made asymmetric [51,52,62]. In contrast, the developing 1/6 FQHS in our sample is robust against asymmetry (see Figs. S10 and S11 [28]). Fourth, at a fixed QW width, by increasing n, or equivalently $\tilde{w}/l_B$, the ground state at $\nu = 1/2$ in wide QWs starts as a compressible Fermi sea, transitions to an incompressible FQHS, and eventually turns into an insulating phase [51,53,62]. At $\nu = 1/6$, however, the 2DES is deep in the insulating regime, and a developing FQHS emerges when $\tilde{w}/l_B$ becomes large.

It is worth reiterating that, while the origin of the 1/2 FQHS in wide QWs is not entirely clear, recent experiments [54–56] and theories [59,61] point to a single-component, non-Abelian, Pfaffian-like state stabilized by a p-wave pairing of ^{2}CFs. Similarly, the 1/4 FQHS has been interpreted as a single-component, non-Abelian FQHS stabilized

by f-wave pairing of ^{4}CFs [60,61]. Despite the differences enumerated in the previous paragraph between the parameters of our samples and those that show the $1/2$ and $1/4$ FQHSs, we suggest that the developing $1/6$ FQHS we report here is also single-component and non-Abelian, and is stabilized by a pairing of ^{6}CFs [see Fig. 1(d)]. An alternative, two-component ground state, such as the Halperin-Laughlin Ψ_{993} state [63], is extremely unlikely to be the ground state, given the single-layer-like charge distribution in our samples and the persistence of the $1/6$ R_{xx} minimum as we make the charge distribution significantly asymmetric. Consistent with our conjecture that the $1/6$ FQHS we are observing has a one-component origin, preliminary theoretical calculations indeed suggest that this state likely emerges from an f-wave pairing of ^{6}CFs and hosts non-Abelian quasiparticles [64]. We hope, of course, that our results will stimulate future theoretical calculations to shed more light on the origin of the FQHSs at $\nu = 1/6$ (and $1/8$).

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[27] In Fig. 1(a), to measure high R_{xx} (~MΩ), a small driving ac current (0.2 nA) with low frequency (0.05 Hz) was used to avoid heating and nonlinear effects, and to maximize the in-phase (resistive) component of the signal.

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[40] In our samples, the extremely high values of R_{xx} (~MΩ) preclude us from measuring the Hall resistance (R_H) accurately at small fillings $\nu < 1/5$; see SM (Sec. VII, Figs. S12–S14) for Hall data and a detailed discussion [28]. The very large R_{xx}/R_H causes significant mixing of R_{xx} in measurements of R_H. This is a well-known problem for small ν regime where an insulating behavior is dominant [23,25,37,38].

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